On the Cost of Regulation under Solvency II

Carole Bernard, University of Waterloo

An Chen, University of Amsterdam & Netspar

Antoon Pelsser, University of Amsterdam & Netspar

Abstract: This paper shows that insurers can meet solvency requirements without a great sacrifice to the expected return either of themselves or their policyholders and thus that the regulation through Solvency II is not very *"costly"*.

Introduction

Regulatory authorities want to implement a new solvency framework in Europe, Solvency II. In order to meet the solvency requirement of Solvency II, life insurance companies and pension funds fear not to be able to invest much in the equity market under the regulation of Solvency II and to be forced to follow quite conservative risk management strategies. If this is the case, the cost of regulation subject to Solvency II is very high. Our objective is to investigate the validity of this argument. Solvency II advocates risk–based regulation which focuses on *downside risk*. It is likely to use measures such as the ruin probability or the Value-at-Risk. Hence, we assume regulators aim to control the default probability of insurance companies.

Our methodology consists in modeling an insurer who writes specific life insurance equity-linked contracts (which are used for the purpose of illustration only) at time 0. The insurer invests then the premiums with its own equity in a single fund. Regulatory authorities are continuously monitoring the level of the assets of the insurer and watching at a solvency barrier. If the solvency trigger is hit before the maturity of the contracts, then the company is liquidated and the proceeds paid to the policyholder. If the company remains solvent, a rule determines the share-out between the insurer and policyholders. The methodology can be extended to other frameworks.

Using a simple strategy for the insurer and the regulator, we show with a few regulations regulators can significantly reduce the probability of ruin without reducing too much expected returns of the insurer and policyholders. The insurer might sometimes be forced (by the regulator) to switch to a low-risk strategy when they are close to their solvency limits, but not at other times. It hence shows that insurers can meet requirements without a great sacrifice to the expected return either of themselves or their policyholders and thus that the regulation through Solvency II is not very *"costly"*. In this sense, the "cost of regulation" refers to the fact that "the two parties are forced to accept an inferior payoff distribution".

In the following, we first present the model setup and then illustrate conclusions with some numerical examples.

Model setup

We now introduce the model setup. We model an insurance company subject to default risk and asset risk. Default is triggered by the observation of the firm's assets. This model was first applied to insurance by Briys and de Varenne (1994) (no premature default possible) and Grosen and Jørgensen (2002) (premature default possible). The framework is the one by Grosen and Jørgensen (2002), the default occurs when assets drop below liabilities which are modeled by an exponentially increasing barrier level. Consider an insurer operating on the time horizon $[0, T]$. At time 0, the insurer issues a participating equity-linked contract to a representative policyholder who pays an upfront premium P_0. The insurer also receives an amount of initial equity contributions E_0 at time 0. Consequently, the initial asset value of the insurer is given by $A_0 = P_0 + E_0$. From now, we shall denote $P_0 = \alpha A_0$ with $\alpha \in (0,1)$. The initial capital structure of the insurer is summarized in the table below:

Asset	Liability	Equity
A_0	$P_0 = \alpha A_0$	$E_0 = (1-\alpha)A_0$

For simplicity, we assume that the insurer's firm value A_t evolves according to the following geometric Brownian motion:

$$\frac{dA_t}{A_t} = \mu dt + \sigma dW_t, \tag{1}$$

where W_t is a standard Brownian motion. μ and σ are respectively the constant instantaneous rate of return and the volatility of the firm's assets.
As a compensation to their initial investments P_0 and E_0, the policy- and equity holder acquire a claim on the firm's assets at or before maturity T depending on the insurer solvency status.
If liquidation does not occur on $[0, T]$, we denote by $\psi_P(A_T)$ the total payoff to the policyholder at maturity T and by $\psi_E(A_T)$ the total payoff received by the equity older at maturity T. One has:

$$\psi_E(A_T) + \psi_P(A_T) = A_T. \tag{2}$$

The total payoff to the policyholder at maturity T is denoted by $\psi_P(A_T)$ is given by:

$$\psi_P(A_T) = \begin{cases} A_T & \text{if } A_T < P_T \\ P_T & \text{if } P_T \le A_T \le \frac{P_T}{\alpha} \\ P_T + \delta(\alpha A_T - P_T) & \text{if } A_T > \frac{P_T}{\alpha} \end{cases}$$

where

$$P_T = P_0 e^{gT}$$

corresponds to the guaranteed minimum payment at maturity where g is the guaranteed minimum rate of return. If $A_T < P_T$, the company is bankrupted at maturity T. The priority of policyholders implies that they get the full remaining asset's value. Here, $\delta(\alpha A_T - P_T)$ represents the bonus payment to the policyholder as a fraction of the residual surplus adjusted by the policyholder's share α in the insurer's initial capital and a participation rate δ. This bonus is paid if the company has enough benefits. This payoff is depicted in Figure 1 and can also be rewritten as:

$$\psi_P(A_T) = P_T + \delta(\alpha A_T - P_T)^+ - (P_T - A_T)^+$$

where the bonus payment appears to be a call option [1] and the short put option $-(P_T - A_T)^+$ comes from the equity holder's limited liability. It is assumed that the equity holder receives at maturity T

$$\psi_E(A_T) = (A_T - P_T)^+ - \delta\left(\alpha A_T - P_T\right)^+. \tag{3}$$

This payment reflects the fact that the equity holder has a lower claim priority on the insurer's residual assets than the policyholder.

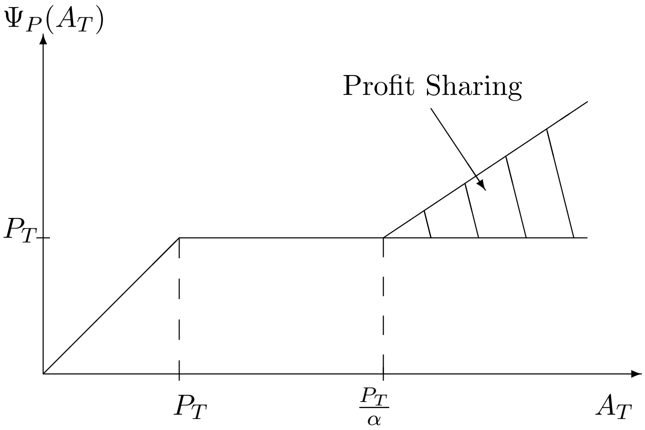


Figure 1: The payoff $\Psi_P(A_T)$ to the policyholder given no premature liquidation

We now turn to the case when a liquidation of the insurer is enforced by the regulator. We suppose that the regulator monitors the firm's assets value A_t continuously because a company has to be solvent at any time[2]. Default and liquidation[3] are carried out by the regulator when the insurer's firm assets A_t become too low, mathematically when they hit some deterministic time-dependent barrier B_t:

$$B_t = \eta P_t \tag{4}$$

[1] We use $(x)^+$ to denote $\max(x, 0)$.

[2] In particular, in the case of a continuous monitoring by the regulator, when policies also include surrender options, the company should be able to give back the promised amount at any time.

[3] For instance, in the US Bankruptcy Code in the Chapter 7 Bankruptcy Procedure, default leads to an immediate liquidation.

for $t \in [0,T]$. We assume $\eta \leq 1$, this parameter η may be regarded as a regulation parameter controlling the strictness of the regulation rule. The liquidation time τ is given by

$$\tau \quad = \quad \inf\{\ t \in [0,T] \mid A_t \leq B_t\ \}. \tag{5}$$

If $\tau < T$, a premature liquidation results. The liquidation date is constructed as the first time of the firm's asset hitting the barrier from above. Upon premature liquidation, a rebate payment

$$\Theta_P(\tau) \quad = \quad B_\tau \tag{6}$$

is provided to the policyholder respectively. Equity holders receive $\Theta_E(\tau) = 0$. Please note that some costs might be added upon liquidation for instance by introducing an additional parameter η_2. Upon liquidation, the policyholders receive only a percentage of the remaining asset $L_t(\eta) = \eta_2 B_t(\eta)$ (instead of $B_t(\eta)$). The parameter η_2 corresponds to the recovery rate. This has already been noticed in previous literature (see for instance Bernard et. al (2006)). It adds a new parameter in the model but does not change our results substantially. The amount to be given to policyholders is lower, and thus the expected return will be lower but it does not change significantly the main message of the paper. We thus voluntarily omit liquidation costs and other types of costs.

In the above setting, the volatility of the firm's assets is assumed to be constant. This implies that the insurance company does not readjust its risk management strategy throughout the contract period and the regulator stays passive. The only intervention time of the regulator is to enforce the liquidation of the insurer. Throughout the paper, this setup is referred to the "static framework". In particular, the insurance company follows a risk management strategy with a fixed volatility.

Default Probability in the Static Framework

Apparently, in the static framework, the default/liquidation probability is characterized by the probability that the firm's assets have hit or fallen below the barrier before the maturity date, i.e.

$$\Pr(\tau \leq \mathrm{T}) = \Pr\left(\inf_{\mathrm{t}\in[0,\mathrm{T}]} \{\mathrm{A_t} \leq \mathrm{B_t}\} \ < \ \mathrm{T}\right). \tag{7}$$

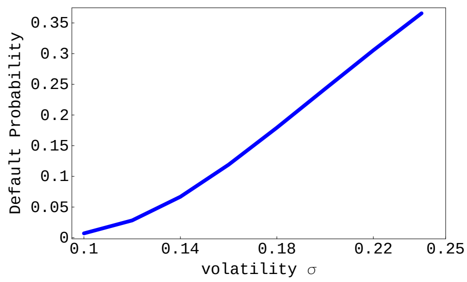


Figure 2: Probability w.r.t. volatility σ. Parameters are set to $r = 5\%$, $\alpha = 0.8$, $A_0 = 100$, $P_0 = 80$, $T = 20$ years, $g = 2\%$ and $\eta = 0.8$ and μ is chosen to satisfy $(\mu - r)/\sigma = 0.2$.

In this framework, this probability can be computed explicitly (c.f. Bernard and Chen (2008)). The objective of the regulator is to constrain this probability to stay under a maximum allowed probability constraint ε. On Figure 2, we represent this probability with respect to the volatility σ, we observe a bijection between this probability and the volatility.

Expected annual log-return

Equity holders are more interested in their expected return than in the default probability. We define the expected annual log-return of the policyholder as:

$$PER = \frac{1}{T}\ln\left(\frac{\text{Expected Payoff at time } T}{P_0}\right),$$

where the expected payoff is given by

$$E\left[\psi_P(A_T)\mathbb{1}_{\{\tau>T\}}\right] + E\left[\Theta_P(\tau)e^{r(T-\tau)}\mathbb{1}_{\{\tau\leq T\}}\right].$$

$\mathbb{1}_{\{x\}}$ denotes the indicator function of an event x and it gives the value 1 if x holds and else 0. The rebate payment $\Theta_P(\tau)$ is paid at the liquidation time τ and is accumulated with the risk free interest rate r to the maturity date T for time consistency reasons. Similarly, the expected annual log-return of the equity holder is given by:

$$EER = \frac{1}{T}\ln\left(\frac{\text{Expected Payoff at time } T}{E_0}\right),$$

where the expected payoff of the equity holder writes as:

$$E\left[\psi_E(A_T)\mathbb{1}_{\{\tau>T\}}\right].$$

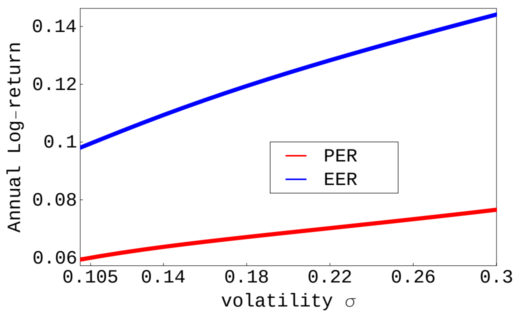


Figure 3: PER and EER w.r.t. volatility σ.
Parameters are set to $r = 5\%$, $\alpha = 0.8$, $A_0 = 100$, $P_0 = 80$, $T = 20$ years, $g = 2\%$ and $\eta = 0.8$ and μ is chosen to satisfy $(\mu - r)/\sigma = 0.2$.

We need to define the participation rate δ with which the policyholder is allowed to participate in the surpluses of the insurance company. For the simulation, δ is set at a percentage of the fair participating rate $\delta = 0.9\,\hat{\delta}$. $\hat{\delta}$ denotes the *fair* participation rate, that is the initial investment of the policyholder is equal to the market value of the acquired claim[4]. Following the ideas of Boyle and Tian (2007), we use a δ-value lower than the fair value to take account of the safety loading in the pricing of equity-linked life insurance.

A straightforward observation from Figures 2 and 3 is that the higher the volatility, the riskier the strategy and the higher the expected annual log-returns PER and EER of the strategy. Note that the expected annual log-return for an investment in the risk-free rate is equal to 0.05. Since equity holder bear more risk and the policyholder, it is observed that PER is slightly, whereas EER is substantially greater than 5%.

Volatility-switching model

The static framework might be reasonable for short–term contracts, but the life insurance contracts considered here are often long–term contracts with a maturity T equal to 10 to 20 years. During such a long-term contract period, it is very likely that the insurance company follows a risk management strategy with a non-constant volatility. For instance, the insurance company might readjust its risk management strategy, by periodically switching to different volatilities, in order to avoid the regulator's intervention. We use "dynamic framework" to describe this setting. Both the regulator and the insurance company do not stay passive during the contract period. First, the insurer can react to the regulator's rule and adjust his risk management strategy. Second, the regulator can intervene and force the insurance company to switch to a less risky strategy when the default probability exceeds the probability constraint.

In the following, we introduce a very simple strategy where the insurance company can only choose between two portfolios with different volatilities. Although such a simple strategy is considered for purpose of illustration, we believe the strategy is quite representative. As proposed by Dangl and Lehar (2004) for a bank, we assume that there are two different portfolios with two levels of assets risk: σ_L and σ_H corresponding to two instantaneous return rates μ_P and μ_H. We assume both the insurer and the regulator can take actions at a discrete set of dates (for instance at the end of each year)[5]:

$$\underline{\underline{T}} \quad = \quad \{t_0, t_1, \cdots, t_{N-1}\} \tag{8}$$

with $t_N := T$. At each date t_i before the maturity of the contract, four different events might occur:

1) The regulators look at the value of the assets of the company and declare bankruptcy because it is too low.
2) The company is solvent but too risky: regulators force the company to switch the level of asset risk to a lower level.
3) Given the regulatory requirements, it is optimal for the managers to stick to the current risk level.
4) Given the regulatory constraints, it is optimal to switch the level of asset risk. In that case that

[4]The fair participation rate $\hat{\delta}$ results from the fair valuation principle, i.e.

$$E^*[e^{-rT}\left(\hat{\delta}[\alpha A_T - P_T]^+ + P_T - [P_T - A_T]^+\right)\mathbb{1}_{\{\tau \geq T\}}] = P_0,$$

where E^* represents the expectation taken under the equivalent martingale measure.

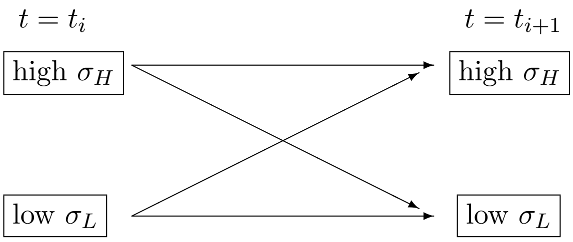
[5]In practice, continuous monitoring is not possible, and regulators observe the insurer's assets at a discrete set of dates.

means the company performs well and can take more risk.

Given a maximum probability ε of bankruptcy before maturity T, then the company wants to maximize the equity holder's value keeping the probability of an early closure below ε. The insurance company switches the portfolio at the end of each year as long as no early default occurs. At the end of each year $t = t_i$ $(i = 1, \cdots, T)$, managers face three different situations:

- *Case 1:* $A_t < B_t$ Bankruptcy is declared, equity holders receive nothing and policyholders receive A_t.
- *Case 2:* $A_t \geq B_t$ and $\sigma = \sigma_H$. We then compute at time t, the probability of bankruptcy before T when there is no switching until T (according to Equation (7)). If this default probability is above ε, then regulators reduce the level of the volatility, otherwise they do not intervene.
- *Case 3:* $A_t \geq B_t$ and $\sigma = \sigma_L$. The managers decide to switch to a higher risk level in order to increase their expected payment. Their decision should keep on satisfying that bankruptcy probability before maturity is below ε.

The following scheme elucidates the above strategy which provides the possibility to switch between the high and low risk management strategies.



Expected return in the Dynamic Framework

If a dynamic risk management strategy leads to a reduction in the probability of liquidation but simultaneously to a substantial reduction in the expected return, this strategy might be undesirable. The reduction in the expected returns accompanying a reduction of the liquidation probability can be interpreted as the "cost of regulation", when the regulation objective is set by controlling the liquidation probability. Phrasing it in another way, the "cost of regulation" implies that "the two parties are forced to accept an inferior payoff distribution" and is indeed the opportunity cost from the loss in expected return due to the actions of the regulator. We want to show that insurers can meet requirements without a too great sacrifice.

In the next section, we mainly examine whether regulation is so "costly" that the insurance company following Solvency II shall not invest in the risky asset at all. In fact, we show that the dynamic approach can lead to promising results, namely a lower probability of liquidation accompanied with an acceptable reduction in expected returns.

Comparative statistics

In this section, we carry out a numerical analysis to first present the results for the static setting, from which we figure how high the cost of regulation is when the insurance companies or pension funds give up investing in high-risky equity completely. We then move to the dynamic setting and compare the results with the static setting. We observe that the regulation is indeed not very "costly" when the insurance companies or pension funds follow a dynamic risk management strategy, which is certainly a realistic assumption for a long time horizon.

Risk and returns in the static setting

We set interest rate at 5%, the maturity T of the contract at 20 years. The initial assets' value is equal to $A_0 = 100$. The ratio invested by policyholders is 80%. The intervention level at time $t \in [0, T]$ is given by

$$B_t = 0.8 P_t$$

where $P_0 = \alpha A_0 = 80$ and $g = 2\%$. In the dynamic setting, there are two possible investments. The low risk investment is such that $\mu_L = 6\%$ and $\sigma_L = 5\%$. The riskier investment is characterized by $\mu_H = 9\%$ and $\sigma_H = 20\%$ [6]. The participating coefficient is decided at the beginning with the volatility level at

[6] The choice of μ_L, σ_L and that of μ_H, σ_H lead to the same Sharpe ratio.

time 0 such that $\delta = 0.9\,\hat{\delta}$. When $\sigma_0 = 5\%$, then $\delta = 89.96\%$, when $\sigma_0 = 20\%$, then $\delta = 75.26\%$.

In a static setting, we are able to compute the probability of a regulators' closure decision before the maturity T in case of continuous monitoring if the initial assets risk is set at σ_0 (c.f. Equation (7) and Figure 2). Furthermore, the expected annual log-returns for both the policy and equity holder can be calculated explicitly (see also Figure 2). Table 1 provides some results for both the high and low risky asset case. It is observed that a cumulative default probability of 29.3% results if the insurance companies or pension funds invest in a high risky asset ($\sigma_0 = \sigma_H$) over the entire time horizon (here 20 years). Apparently it is a very high and unrealistic default probability. In order to achieve a reasonable cumulative default probability, assume that the insurance companies or pension funds are forced to invest a low risky asset, i.e. $\sigma_0 = \sigma_L$ throughout the operating time. By giving up investing in the high risky asset completely, the default probability is reduced substantially (close to zero). However, it leads to quite big reductions in the expected annual log-returns at the same time (a reduction of 24.1% for the policyholder and of 32.1% for the equity holder), which illustrate the cost of regulation. This is exactly the concern of the insurance companies or pension funds, as stated in Gollier (2008). In order to meet the solvency requirement of Solvency II, they are forced (by the regulator) to reduce the relative riskiness of equity (particularly for long time horizon), which results in a substantial reduction of expected returns for both the policy and equity holder.

	$\sigma_0 = \sigma_L = 5\%$ $\mu_L = 6\%$	$\sigma_0 = \sigma_H = 20\%$ $\mu_H = 9\%$
Default Prob.	$9.35 \cdot 10^{-5}\%$	29.3%
PER	8.8%	11.6%
EER	10.6%	15.6%

Table 1: Liquidation probability and expected annual log-return in a static setting for policy and equity holder.

Risk and returns in the volatility–switching model

For a long time horizon, it is very likely that the insurance companies or pension funds readjust their risk management portfolios (to another volatility). This subsection provides some simulation results for the volatility-switching strategy model described above. The portfolio is readjusted in order to maximize the expected returns to the policy and equity holder keeping a liquidation probability prior to maturity below a maximum level. Here, we fix a maximum level of risk (through a given liquidation probability ε e.g. 2%).

Table 2 illustrates the default probability and the expected annual log-returns for the policy and equity holder in a dynamic setting where the initial riskiness of the equity is given by 20%. Compared to the static setting ($\sigma_0 = \sigma_H$), firstly, a considerably lower default probability (now 0.017%) is observed. Although this probability is higher than the one obtained in the static setting ($\sigma_0 = \sigma_L$), it is a very acceptable level of default probability for a 20-year time horizon. Secondly and more importantly, the reductions in the expected annual log-returns are much lower. Compared to the case where the insurance companies or pension funds give up investing in high risky equity completely, the equity holder "suffers" much less. The level of reduction is much less substantial (11.5% vs 32.1%.) Whereas for the policyholder, the level of reduction is less pronounced (16.4% vs 24.1%). In other words, the dynamic setting has a consequence that it decreases significantly the default probability keeping rather interesting expected returns.

From the regulator's viewpoint, the impact of the volatility-switching model is extremely interesting. In a dynamic setting, the insurance company might be forced to reduce the risk level of its risk management strategy. This is exactly what most of insurers worry, i.e. they fear not to be able to trade in risky assets at all. However, the readjustment to a less risky portfolio will be very *temporary*. For instance, provided that in the following period, the cumulative liquidation probability for the residual time is lower than the maximal allowed one (and the firm's asset still lives above the barrier), the insurance company can switch back to high risky asset. This argument is indeed verified by observing the relatively small reduction of the expected returns. It implies that the regulation (to satisfy the solvency requirement) is in fact not very "costly" as most worry or overstate. Under Solvency

II regulation, the insurance company might invest a bit conservatively temporarily, it is certainly not true that the company shall invest very conservatively throughout the contract period.

	$\sigma_0 = \sigma_H = 20\%$ $\mu_H = 9\%$
Default Prob.	0.017%
PER	9.7% (-16.4%)
EER	13.8% (-11.5%)

Table 2: Default probability and expected payments for the policy and equity holder in case of dynamic approach. In parenthesis, we give the percentage of increase or decrease compared to the situation with static case.

Conclusion

In this paper, we establish a simple volatility–switching model to describe the interaction between the insurer and the regulator. We show the regulation along the lines of Solvency II does not necessarily lead to the consequence that the insurance company has to invest very conservatively.

References

[1] Bernard, C., Chen, A., 2008. On the regulator-insurer-interaction in a structural model. *IIPR Insurance and Pension Techinical Report* 2008-01.

[2] Bernard, C., Le Courtois, O., and Quittard-Pinon , F, 2006. Development and pricing of a new participating contract. *North American Actuarial Journal*, 10(4), 179–195.))

[3] Boyle, P., Tian, W., 2008. The design of equity-linked contracts. Forthcoming in *Insurance: Mathematics and Economics.*

[4] Briys, E., de Varenne, F., 1994. Life insurance in a contingent claim framework: pricing and regulatory implications. *Geneva Papers on Risk and Insurance Theory* 19(1), 53–72.

[5] Dangl, T., Lehar, A., 2004. Value-at-risk vs. building block regulation in banking. *Journal of Financial Intermediation* 13, 96–131.

[6] Gollier, C., 2008. Assets relative risk for long-term investors. *Forthcoming in Life and Pensions.*

[7] Grosen, A., Jørgensen, L., 2002. Life insurance liabilities at market value: an analysis of insolvency risk, bonus policy, and regulatory intervention rules in a barrier option framework. *Journal of Risk and Insurance* 69(1), 63–91.