Optimal Reinsurance Arrangements Under Tail Risk Measures

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Abstract

Regulatory authorities demand insurance companies to control their risk exposure by imposing stringent risk management policies. This article investigates the optimal risk management strategy of an insurance company subject to regulatory constraints. We provide optimal reinsurance contracts under different tail risk measures and analyze the impact of regulators' requirements on risk-sharing in the reinsurance market. Our results underpin adverse incentives for the insurer when compulsory Value-at-Risk risk management requirements are imposed. But economic effects may vary when regulatory constraints involve other risk measures. Finally, we compare the obtained optimal designs to existing reinsurance contracts and alternative risk transfer mechanisms on the capital market.

Keywords: Optimal Reinsurance, Risk Measures, Alternative Risk Transfer

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Introduction

European insurers have recently experienced increasing stress to incorporate the strict capital requirements set by Solvency II. One key component of Solvency II is to determine the economic capital based on the risk of each liability in order to control the probability of bankruptcy, equivalently, the Value-at-Risk [1]. This paper examines the optimal risk management strategy when this type of regulatory constraint or alternative risk constraints are imposed to the insurer. We show that if the insurer minimizes the insolvency risk, an optimal strategy is to purchase a reinsurance contract to insure moderate losses but not large losses. Therefore, Value-at-Risk could induce adverse incentives to insurers not to buy insurance against large losses. The same strategy is also optimal when the insurer wants to minimize the Conditional Tail Expectation (CTE) of the loss[2]. However, the optimal reinsurance contract is a deductible when the insurer minimizes the expected variance. Hence, the optimal risk management policy varies in the presence of different risk measures.

Our results offer some economic implications. *First*, our results confirm that regulation may induce risk averse behaviors of insurers and increase the reinsurance demand. Mayers and Smith (1982) were the first to recognize that insurance purchases are part of firm's financing decision. The findings in Mayers and Smith (1982) have been empirically supported or extended in the literature. For example, Yamori (1999) empirically observes that Japanese corporations can have a low default probability and a high demand for insurance. Davidson, Cross and Thornton (1992) show that the corporate purchase of insurance lies in the bondholder's priority rule. Hoyt and Khang (2000) argue that corporate insurance purchases are driven by agency conflicts, tax incentives, bankruptcy costs and regulatory constraints. Hau (2006) shows that liquidity is important for property insurance demand. Froot, Sharfstein and Stein (1993), Froot and Stein (1998) explain the firm would behave risk averse because of voluntary risk management.This paper contributes to the extensive literature aiming to explain why risk neutral corporations purchase insurance. As shown in this paper, risk neutral insurers may behave as risk averse agents in the presence of regulations.

Second, our results provide some rationale of the conventional reinsurance contracts and link existing reinsurance contracts with derivative contracts available in the capital market. Froot (2001) observes that "*most insurers purchase relatively little cat reinsurance against large events*". Froot (2001) shows that "excess-of-loss layers" are however suboptimal and that the expected utility theory can not justify the capped features of the reinsurance contracts in the real world. Several reasons for these departures from the theory are presented in Froot (2001). Our paper partially justifies the existence of "excess-of-loss layers" from a risk management perspective. When the insurer implements risk management strategies based either on the VaR or the CTE, the insurer is not willing to hedge large losses. Therefore, the optimal risk management strategy

[1] For the current stage of Solvency II we refer to extensive documents in http://www.solvency-2.com.

[2] The idea of CTE is to capture not only the probability to incur a high loss but also its magnitude. From a theoretical perspective CTE is better than VaR (see Artzner et al. (1999), Inui and Kijima (2005)), and it has been implemented to regulate some insurance products (with financial guarantees) in Canada (Hardy (2003)).

involves insuring moderate losses more than large losses, which is consistent with the empirical evidence of Froot (2001).

Third, our results offer some risk-sharing analysis in the reinsurance market. This analysis and the methodology could be helpful for both insurers and insurance regulators to compare the effects of imposing different risk constraints on insurers and to investigate which risk measure is more appropriate.

This study is organized as follows. The next section describes the model and we derive the optimal reinsurance contract under the VaR risk measure. The following section solves the optimal reinsurance problems under other risk measures. Then we compare the optimal reinsurance design with previous literature in which the firm behaves risk averse in other frameworks. We finally compare the optimal reinsurance contracts under risk measures to contracts frequently sold by reinsurers in the marketplace. The final section summarizes and concludes the study. Proofs are given in appendix.

1 Optimal Reinsurance Design under VaR Measure

We consider an insurance company with initial wealth W_0, which includes its own capital and the collected premia from sold insurance contracts. Its final wealth, at the end of the period, is $\widehat{W} = W_0 - X$ if no reinsurance is purchased, where X is the aggregate amount of indemnities paid at the end of the period. The insurer is assumed to be risk-neutral and faces a risk of large loss.

We assume that regulators require the insurer to meet some risk management requirement. As an example, assume that ν is a Value-at-Risk (VaR) limit to the confidence level α, then the VaR requirement for the insurer is written as:

$$\mathbb{P}\left\{W_0 - \widehat{W} > \nu\right\} \leqslant \alpha. \tag{1}$$

This probability $\mathbb{P}\{W_0 - \widehat{W} > \nu\}$ measures the *insolvency risk*. This type of risk management constraint has been described explicitly in Solvency II.[3]

In a reinsurance market, the insurer purchases a reinsurance contract with indemnity $I(X)$ from a reinsurer, paying an initial premium P. If a loss X occurs, the insurance company's final wealth becomes $W = W_0 - P - X + I(X)$. $I(X)$ is assumed to be non-negative and can not exceed the size of the loss. The final loss L of the insurance company is $L = W_0 - W = P + X - I(X)$, a sum of the premium P and the retention of the loss $X - I(X)$. The VaR requirement (1) is formulated as $\mathbb{P}\{L > \nu\} \leqslant \alpha$. It is equivalent to $VaR_L(\alpha) \leqslant \nu$ [4].

[3] Both parameters ν and α are often suggested by regulators.

[4] VaR is defined by $VaR_L(\alpha) = \inf\{x,\ \mathbb{P}\{L > x\} \leqslant \alpha\}$.

We assume the following premium principle P:

$$P = \mathbb{E}[I(X) + C(I(X))] \tag{2}$$

where the cost function $C(.)$ is non-negative and satisfies $C'(\cdot) > -1$. Note that this assumption is fairly general and include many premium principles as special cases. For instance, Arrow (1963) assumes that the premium depends on the expected payoff of the policy only. Gollier and Schlesinger (1996) consider a similar premium principle. Raviv (1979) considers a convex cost structure while Huberman, Mayers and Smith (1983) introduce a concave cost structure. The objective in this section is to search for a optimal indemnity $I(X)$ under the constraint (1).[5] Precisely,

Problem 1.1 *Find a reinsurance contract $I(X)$ that minimizes insolvency risk:*

$$\min_{I(X)} \ \mathbb{P}\{W < W_0 - \nu\} \quad s.t. \quad \begin{cases} 0 \leqslant I(X) \leqslant X \\ \mathbb{E}\left[I(X) + C(I(X)\right] \leqslant \Delta \end{cases} \tag{3}$$

In Problem 1.1 the probability $\mathbb{P}\{W < W_0 - \nu\}$ can be written as an expected utility $\mathbb{E}[u(W)]$ with a utility function $u(z) = \mathbb{1}_{z<W_0-\nu}$. This utility function, however, is not concave. Therefore, standard Arrow-Raviv first-order conditions (see Arrow (1963, 1971), Borch (1971), and Raviv (1979)) are not sufficient to characterize the optimum. This remark also applies to subsequent problems investigated in the paper.

Problem 1.1 can be motivated as follows. Note that $\mathbb{E}[W] = W_0 - \mathbb{E}[X] - P + \mathbb{E}[I(X)] = W_0 - \mathbb{E}[X] - \mathbb{E}[I(X) + C(I(X))] + \mathbb{E}[I(X)]$. Therefore,

$$\mathbb{E}[I(X) + C(I(X))] \leqslant \Delta \quad \Longleftrightarrow \quad \mathbb{E}[W] \geqslant W_0 - \mathbb{E}[X] - \Delta + E[I(X)]. \tag{4}$$

Then Problem 1.1 characterizes the efficient risk-return profile between the guaranteed expected wealth and the insolvency risk measured by the probability that losses exceed the VaR limit. To some extents, Problem 1.1 is similar to a safety-first optimal portfolio problem considered by Roy (1952). The "*safety first*" criterion is a risk management technique that allows you to select one portfolio over another based on the criteria that the probability of the return of the portfolios falling below a minimum desired threshold is minimized. Roy (1952) obtains the efficient frontier between risk and return, measured by the default probability and the expected return, respectively. Figure 2 below displays this efficient frontier in our framework.

Minimizing the insolvency risk under VaR constraint, as stated in Problem 1.1, is important. However, some other issues are not addressed in Problem 1.1. For instance, we ignore the interests of the debtholders (policyholders and bondholders) of the insurer. Even a small probability

[5] By the optimality in this paper we mean a Pareto-optimality. In the optimal insurance literature, there are two separate concepts, one is to determine the optimal shape of the insurance contract and another one is to find the optimal level of insurance. The optimal premium level, which is not addressed in this paper, can be solved via a numerical search. See Schlesinger (1981), Meyer and Ormiston (1999).

of default could lead to a huge loss of the debtholders. Therefore, the objective function in Problem 1.1 is not necessarily optimal to debtholders. The agency problem between the shareholder and the managers is also overlooked in Problem 1.1. It is appropriate to view the objective in Problem 1.1 as an optimal strategy for the managers as the unemployment risk naturally follows from default risk [6]. A more natural problem, from the shareholder's perspective, is to maximize the expected wealth subject to a VaR probability constraint, or subject to a limited liability constraint (Gollier, Koehl and Rochet (1997)). The latter optimal risk management problem in this context is more complicated than Problem 1.1. For example, Gollier, Koehl and Rochet (1997) find that the limited liability firm is more risk-taking than the firm under full liability. The problem of the shareholder is even harder if a probability constraint is imposed in this expected utility framework[7]. Because we confine ourselves to a risk-neutral framework, the discussion of those extended optimal reinsurance problems is beyond the scope of this paper.

Let $\mathcal{S} := \left\{P \,:\, 0 \leqslant P < \mathbb{E}\left[(X-\nu+P)^+ + C\left((X-\nu+P)^+\right)\right]\right\}$. The solution of Problem 1.1 is given in the following proposition.

Proposition 1.1 *Assume X has a continuous cumulative distribution function[8] and $P \in \mathcal{S}$. Let*

$$I_P(X) = (X+P-\nu)^+ \mathbb{1}_{\nu-P\leqslant X\leqslant \nu-P+\kappa_P}, \tag{5}$$

where $\kappa_P > 0$ satisfies $\mathbb{E}\left[I_P(X) + C\left(I_P(X)\right)\right] = P$. Define for $P \in \mathcal{S}$, the probability that a loss exceeds the VaR limit, $\mathbb{L}(P) := \mathbb{P}\left\{X > \nu - P + \kappa_P\right\}$. Then $I_{P^*}(X)$ is an optimal reinsurance contract of Problem 1.1 where P^* solves the static minimization problem

$$\min_{0\leqslant P\leqslant \Delta} \mathbb{L}(P). \tag{6}$$

The proof of Proposition 1.1 is given in Appendix. If, in particular, the VaR limit ν is set to be the initial wealth W_0, then Problem 1.1 is to minimize the ruin probability $\mathbb{P}\{W < 0\}$. This special case is solved by Gajek and Zagrodny (2004). Even though they discover the same optimal shape, their construction of the optimal coverage is not explicit. The same minimal ruin probability problem is also studied in Kaluszka and Okolewski (2008) under different premium principles.

By Proposition 1.1, the optimal reinsurance coverage of Problem 1.1 involves a deductible for small losses, and no insurance for large losses. We call it "*truncated deductible*". This proposition is intuitive appealing. Recall that Problem 1.1 minimizes the insolvency probability. The

[6]The emerging large default risk often leads to loss of confidence of the managers, significant drops of the share price, pressure from directors and shareholders. All these factors make managers worry about their employment status.

[7]We refer to Basak and Shapiro (2001), Boyle and Tian (2007), and Leippold, Trojani and Vanini (2006) for similar problems in finance.

[8]We allow the case when there is a mass point at 0 meaning $\mathbb{P}(X = 0)$ can be positive.

optimal contract must be one in which the indemnity on "*bad*" states are transformed to the "*good*" states by keeping the premium (total expectation) fixed. Because reinsurance is costly, it is optimal not to purchase reinsurance for the large loss states.

An important consequence of the non-concavity of the objective function in Problem 1.1 is that the premium constraint $\mathbb{E}\left[I(X)+C(I(X))\right] \leqslant \Delta$ is not necessarily binding. In other words, the probability $\mathbb{L}(P)$ is not necessarily monotone, hence, the optimal premium P^* does not necessarily satisfy the premium equation $\mathbb{E}\left[I(X)+C(I(X))\right] = \Delta$. This particular feature of the optimal solution of Problem 1.1 is a surprise because, intuitively, the higher premium should lead to smaller solvency probability. This property follows from the specific shape of the truncated deductible indemnity. When the premium P increases, the deductible $\nu - P$ decreases, but the limit $\nu - P + \kappa_P$ could increase or decrease.

To illustrate this notable feature, we present a numerical example as follows. Let $W_0 = 1000$, $\Delta = \nu = \frac{W_0}{10} = 100$, the loss X is distributed with the following density function:

$$f(x) = \frac{2}{ab} x \mathbb{1}_{x \leqslant a} + \left(\frac{2}{b} + \frac{2}{b(b-a)}(a-x) \right) \mathbb{1}_{a < x \leqslant b}, \tag{7}$$

where $a = 25$ and $b = 250$. The premium principle is $P = (1+\rho)\mathbb{E}[I(X)]$ where $\rho = 0.12$ is the loading factor. For $P \in (0, \nu)$, we solve the quantile q so that:

$$\frac{P}{1+\rho} = \mathbb{E}\left[(X - (\nu - P))^+ \mathbb{1}_{X \leqslant q} \right]. \tag{8}$$

Then $\mathbb{L}(P)$ is the probability of X to be more than q. Figure 1 displays a U-shape of the probability function $\mathbb{L}(P)$: $\mathbb{L}(P)$ is first decreasing and then increasing with respect to the premium P. Clearly, the optimum $\min_{0 \leqslant P \leqslant \Delta} \mathbb{L}(P)$ is not equal to Δ. In fact, the minimum is attained when the optimal premium $\mathbb{P}^*$ is approximately equal to 74.

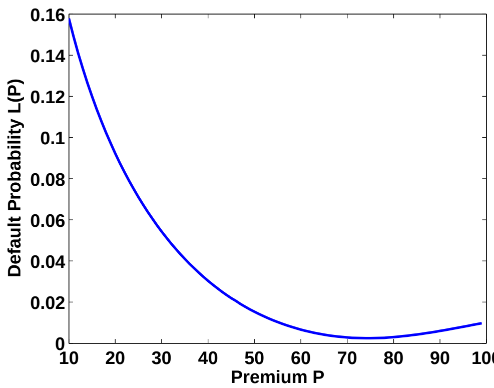

Figure 1: Probability $\mathbb{L}(P)$ w.r.t. P

We now explain why the regulator's risk measure constraint induces risk aversion to risk neutral insurers. In our model it is optimal for a risk neutral insurer not to buy insurance in the absence of regulators. When there exists a regulation constraint, the optimal insurance design is derived as a truncated deductible. Figure 2 displays the trade-off between the expected return (through the final expected wealth) with respect to the risk (measured by the confidence level α). Figure 2 shows that this trade-off shape is concave which resembles the trade-off between risk and return for a risk-averse investor. This figure clearly implies that risk neutral insurance companies react as risk-averse investors in the presence of regulators (this is an *induced risk aversion* [9]).

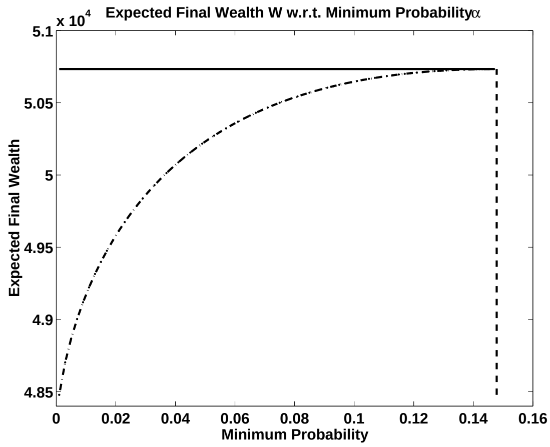

Figure 2: Expected Wealth w.r.t. the probability to exceed the VaR limit
Assume $X = e^Z$ where Z is a Gaussian random variable $\mathcal{N}(m, \sigma^2)$ where $m = 10.4$ and $\sigma = 1.1$. $W_0 = 100,000$, $\rho = 0.15$.

At last, we explain how our result is related to previous literature to finish the discussion of this section. The dual problem of Problem 1.1 is to find the minimal possible premium such that the insolvency probability is bounded by the confidence level α. Therefore, Proposition 1.1 also solves the dual problem by a truncated deductible contract. A variant of this optimal reinsurance problem has recently been studied by Wang et al. (2005). In Wang et al. (2005), the optimal reinsurance contract is the one that maximizes the expected wealth subject to the probability constraint $\mathbb{P}\{W > \mathbb{E}[W] - \nu\} \geqslant 1 - \alpha$, where the premium is $(1+\rho)\mathbb{E}[I(X)]$. Problem 1.1 is significantly different from the problem in Wang et al. (2005) in several aspects. *First*, Wang et al. (2005) consider the deviation from the mean, $W - \mathbb{E}[W]$, while a conventional VaR concept is about the quantile of $W - W_0$. *Second*,

$$W - \mathbb{E}[W] = \quad (I(X) - \mathbb{E}[I(X)]) - (X - \mathbb{E}[X]) \tag{9}$$

[9]Terminology proposed by Caillaud, Dionne and Jullien (2000) in another context.

which is independent of the loading factor ρ. Consequently, the minimal insolvency probability considered in Wang et al. (2005) is independent of the loading factor. Hence Proposition 1.1 can not derived from Wang et al. (2005) [10]

2 Optimal Reinsurance under other Tail Risk Measures

We have explored the optimal reinsurance contract under VaR constraint. Adopting an optimal reinsurance arrangement under VaR constraint, insurance companies choose to leave the worst states uninsured. In this section we derive the optimal reinsurance contracts when other risk measures are imposed. For simplicity of notations, we assume $P = (1+\rho)\mathbb{E}[I(X)]$, where ρ is a constant loading factor. Results of this section can be easily extended to the premium that is a function of the actuarial value of the indemnity.

CTE Risk Measure

The motivation of CTE is to limit the amount of loss instead of its probability of occurrence only. The optimal design problem under the conditional tail expectation can be stated as follows.

Problem 2.1 *Solve the indemnity $I(X)$ such that*

$$\min_{I(X)} \{\mathbb{E}\left[(W_0 - W)\mathbb{1}_{W_0 - W > \nu}\right]\} \quad s.t. \quad \begin{cases} 0 \leqslant I(X) \leqslant X \\ (1+\rho)\mathbb{E}\left[I(X)\right] \leqslant \Delta \end{cases} \tag{10}$$

where the loss level ν is exogenously specified, not related to $(1-\alpha)$-quantile of the loss. Problem 2.1 is consistent with the one investigated in Basak and Shapiro (2001).

Proposition 2.1 *Assuming X has a continuous cumulative distribution function strictly increasing on $[0, +\infty)$. Assume that $P \in \mathcal{S}$, Let*

$$I_P^c(X) = (X + P - \nu)^+ \mathbb{1}_{\nu - P \leqslant X \leqslant \nu - P + \lambda_P} \tag{11}$$

where $\lambda_P > 0$ satisfies that $(1+\rho)\mathbb{E}\left[I_P^c(X)\right] = P$. Let W_P^c be the final wealth derived from $I_P^c(X)$. Then the indemnity $I_{P^*}^c(X)$ solves Problem 2.1, when P^* minimizes:

$$\min_{0 \leqslant P \leqslant \Delta} \mathbb{E}\left[(W_0 - W_P^c)\mathbb{1}_{W_0 - W_P^c > \nu}\right] \tag{12}$$

[10] Moreover, some technical points have been overlooked by Wang et al. (2005) such as the fact that the VaR constraint is binding when X has a continuous distribution which is not true as proved by Figure 1.

The proof of Prop. 2.1 is similar to the one of Prop. 1.1 and omitted. Full details can be obtained from authors upon request. Note that $I_P^c(X)$ is the same as $I_P(X)$ of Prop. 1.1 for a linear premium principle. Then $\lambda_P = \kappa_P$. However the optimal P^* in both Prop. 1.1 and 2.1 are different because of different objective functions in the second stage of the solution. Hence the script "c" is used to denote the CTE constraint for short.

In the presence of a CTE constraint, Prop. 2.1 shows that a risk neutral insurer behaves similarly as under a VaR constraint. Insurers have no incentive to protect themselves against large losses under the conditional tail expectation's constraint. This result seems unexpected because people often argue that CTE is better than VaR (see Basak and Shapiro (2001), Artzner et al. (1999)). The intuition is as follows. In terms of the loss variable $L = W_0 - W$, Problem 2.1 solves

$$\min_L \mathbb{E}[L\mathbb{1}_{L>\nu}] \tag{13}$$

subject to $P \leqslant L \leqslant P + X$. Therefore, the objective is to investigate the trade-off between the amount L and the probability $\mathbb{P}\{L > \nu\}$. Because $\mathbb{E}[L]$ is fixed, the indemnity is small (resp. large) on "bad" states that occur with small probabilities (resp. on "good" states that have a high probability of occurence). The optimal indemnity is to have small loss on "*good*" states, and large loss on "*bad*" states.

Both Prop. 1.1 and Prop. 2.1 are partially consistent with the empirical findings of Froot (2001). Froot (2001) finds that most insurers purchase relatively little reinsurance against catastrophes' risk. Precisely, the reinsurance coverage as a fraction of the loss exposure is very high above the retention (for the medium losses), and then declines with the size of the loss (See Figure 2 in Froot (2001) for details). Arrow's optimal insurance theory implies that this kind of reinsurance contract is not optimal. Froot (2001) provides a number of possible reasons for these departures from theory. Prop. 1.1 and Prop. 2.1 present another explanation of the excess-of-loss layer feature of the reinsurance coverage.

We have mentioned the limitation of Problem 1.1 in last section. Indeed, the model is somewhat too simple to fully explain the design of real insurance contracts regarding large losses. Optimal contracts derived above are also subject to moral hazard since companies might partly hide their large losses. An extension of our model to include the interests of debtholders may be able to overcome these difficulties. In fact, in the presence of asymmetric information, debtholders (policyholders and bondholders) of the insurance company would dislike the contract described in Prop. 1.1 and Prop. 2.1, they would either refuse to participate or require a risk premium to participate. Therefore policyholders ask for a smaller insurance premium and debtholders require larger interests. Hence, the presence of asymmetric information can at least partially justify the fact that insurers purchase coverage for large losses. Since this paper focuses on the effects of the regulatory constraint, we do not model the asymmetric information. Rather, we wonder whether there is any risk measure leading to other type of optimal reinsurance design. In the next subsection, we show that a stronger regulatory requirement may provide incentives to purchase insurance against large losses.

Emphasize the Right Tail Distribution

We now consider a risk measure which based on the expected square of the excessive loss. This risk measure is related to the variance tail measure and thus useful when the variability of the loss is large [11]. Precisely,

Problem 2.2 *Find the optimal indemnity $I(X)$ that solves:*

$$\min_{I(X)} \left\{ \ \mathbb{E}\left[(W_0 - W - \nu)^2 \mathbb{1}_{W_0 - W > \nu}\right] \ \right\} \quad s.t. \quad \begin{cases} 0 \leqslant I(X) \leqslant X \\ (1+\rho)\mathbb{E}\left[I(X)\right] \leqslant \Delta \end{cases} \tag{14}$$

The objective of Problem 2.2 is to minimize the square of excess loss. Comparing with CTE, this risk measure $\mathbb{E}\left[(W_0 - W - \nu)^2 \mathbb{1}_{W_0 - W > \nu}\right]$ pays more attention on the loss amount over the loss states $\{W_0 - W > \nu\}$. Then, it is termed as "expected square of excessive loss measure". [12]

Proposition 2.2 *Assume X has a continuous cumulative distribution function strictly increasing on $[0, +\infty)$ and $P \in \mathcal{S}$. Let d_P be the deductible level whose corresponding premium is P. Then the solution of Problem 2.2 is a deductible indemnity $(X - d_{P^*})^+$, where P^* solves the following minimization problem:*

$$\min_{0 \leqslant P \leqslant \Delta} \mathbb{E}\left[(W_0 - W_P - \nu)^2 \mathbb{1}_{W_0 - W_P > \nu}\right] \tag{15}$$

where W_P is the corresponding wealth of purchasing the deductible $(X - d_P)^+$.

By contrast with Prop. 1.1 and 2.1, Prop. 2.2 states that deductibles are optimal when a constraint on the expected square of excessive loss is imposed. Let us briefly explain why this is the case. In term of the loss $L = W_0 - W$, this problem becomes:

$$\min \mathbb{E}[L^2 \mathbb{1}_{L > \nu}] \tag{16}$$

subject to $\mathbb{E}[L]$ is fixed and $P \leqslant L \leqslant P + X$. In contrast with Problem 2.1, the objective function in Problem 2.2 involves the square of L which dominates the premium constraint $\mathbb{E}[L]$. Then, intuitively, the optimal indemnity should minimize the loss $W_0 - W$ over the bad states as small as possible. Hence the optimal indemnity is deductible. Prop. 2.2 verifies this intuition.

The intuition of Proposition 2.2 can also be found in Gollier and Schlesinger (1996). Gollier and Schlesinger (1996) consider the optimal insurance contract under the second order stochastic dominance approach. By ignoring the "bad scenarios" $\{W_0 - W > v\}$, or when v goes to infinity, Problem 2.2 is in essence to minimize a convex utility, or equivalently, maximize a

[11] For more details on this risk measure we refer to Furman and Landsman (2006).
[12] This measure is not a coherent risk measure in the sense of Artzner et al. (1999).

concave utility. Hence the deducible is optimal when v is extremely high. However, Proposition 2.2 does not follow from Gollier and Schlesinger (1996) directly, because of the non-convex and non-concave feature of $\mathbb{E}[x^2 \mathbb{1}_{x>\nu}]$.[13]

3 Optimal Indemnity with Financing Imperfections

We have shown that risk neutral insurers behave risk averse because of the enforcement of risk measure constraints. The profile of the optimal reinsurance contract depends on how the risk control policy is requested and implemented. Other factors, mentioned earlier in the literature, contribute to the risk averse attitude of risk neutral insurance companies.

Froot, Scharfstein and Stein (1993) consider a value-maximizing company facing financing imperfections increasing the cost of the raising of external funds. The imperfections include cost of financial distress, taxes, managerial motives or other capital market imperfections. Under fairly general conditions on the loss X, Froot, Scharfstein and Stein (1993) prove that the value function $U(W)$ (where W is the internal capital) is increasing and concave. Therefore, the risk neutral firm behaves like a risk averse individual with concave utility function $U(\cdot)$. The deductible indemnity is then optimal (Arrow (1963)). Hence, the optimal reinsurance contract in Froot, Scharfstein and Stein (1993)'s framework is a deductible.

Caillaud, Dionne and Jullien (2000) examine the problem from a different angle by rationalizing the use of insurance covenants in financial contracts, say corporate debts. In Caillaud, Dionne and Jullien (2000), external funding for a risky project can be affected by an accident during its realization. Since accident losses and final returns are private information and can be costly evaluated by outside investors, the optimal financial contract must be a bundle of a standard debt contract and an insurance contract which involves full coverage above a straight deductible. Hence, small loss is not insured because of the auditing costs and the bankruptcy costs.

Figure 3 displays the optimal insurance contract based on either regulatory constraints or costly external funding. On Figure 3, the two indemnities have the same actuarial value, thus the same premium. The truncated deductible indemnity is optimal for VaR or CTE constraints, while the deductible indemnity is optimal for either the square of the expected loss risk measure or voluntary risk management.

The above mentioned literature show that external financing generates insurance demand by risk neutral firms. This amounts to comparing effects of enforcement of regulatory risk constraints and voluntary risk management (to increase firm value). We now compare the risk measure constraint and the voluntary risk management policy. In all possible cases, small losses stay uninsured. But there are significant differences on medium losses and large losses. VaR or a CTE constraints are not enough to induce insurers to protect themselves against large loss amounts. Strong risk control such as the square of the expected loss risk measure provide

[13]The proof is similar to that of Prop. 1.1. Full details to prove Prop. 2.2 can be obtained from authors upon request.

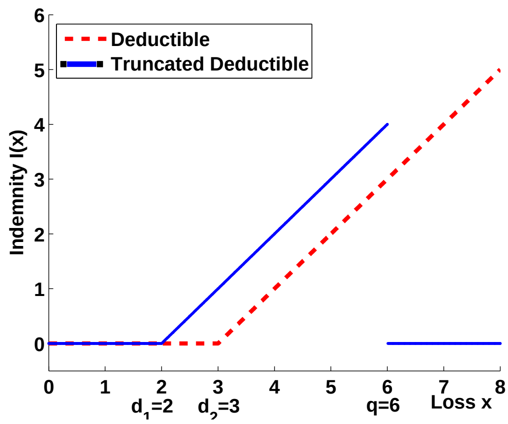

Figure 3: Comparison of reinsurance contracts.
Reinsurance indemnity I w.r.t. the loss X

incentive to insured large loss, hence the optimal indemnity under this risk measure is identical with the one under voluntary risk management policy.

Regulatory requirement and firm's risk management policy lead to different protection (or hedging) strategy. VaR and CTE risk management policies provide a better protection on moderate losses. If the company only implement the enforced constraint without doing a risk averse risk management, it will benefit on average until a large loss occurs. The enforcement of VaR and CTE regulations will be efficient only in the presence of an additional voluntary firm's risk management program.

4 Reinsurance and Capital Market

We now look at the traditional reinsurance policies in the market place. In this section, we first compare our results to traditional reinsurance policies, then interpret reinsurance arrangements as a derivatives portfolio written on a loss index.

We find that the optimal reinsurance contract is not available in the reinsurance market due to moral hazard issues. However, the optimal strategy under VaR can possibly be implemented in the capital market, as soon as a reference index strongly correlated to the insurer's loss is traded.

Froot (2001) underlines that most reinsurance arrangements are "excess-of-loss layers" with a retention level (the deductible level that losses must exceed before coverage is triggered), a limit (the maximum amount reimbursed by the reinsurer) and an exceeding probability (probability losses are above the limit). The contract is written as:

$$I_1(X) = (X - d)^+ - (X - l)^+. \tag{17}$$

where d is the deductible level, l stands for the upper limit of the coverage. This kind of contract is typical by involving a stop loss rule with an upper limit on coverage [14].

In the presence of regulatory VaR or CTE requirements, the optimal reinsurance arrangement is a truncated deductible, $I(X) = (X-d)\mathbb{1}_{X\in[d,q]}$ where d is the deductible level and q the upper limit. Then the indemnity could be expressed as:

$$I(X) = (X-d)^+ - (X-q)^+ - (q-d)\mathbb{1}_{X>q}. \tag{18}$$

Figures 4 illustrates a comparison of these two designs with arbitrary parameters.

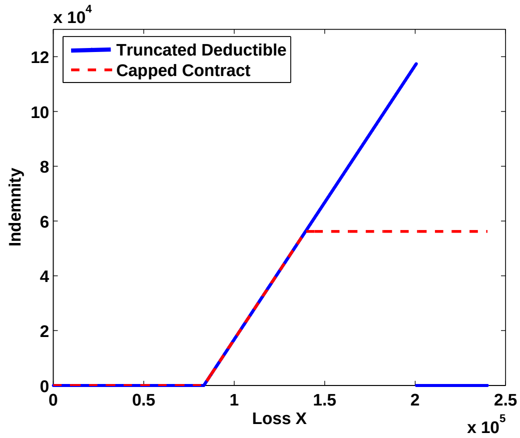

Figure 4: Indemnity $I(X)$ w.r.t. X

We compare the optimal contract and the deductible with an upper limit. On Figure 4, the plain line corresponds to the optimal contract under a VaR constraint and the dash line is the capped contract. They have the same premium P, so the coverage provided by the optimal contract is better for moderate losses but worse for extreme losses.

The difference between the indemnity $I_1(X)$ and $I(X)$ is an indemnity $(q-d)\mathbb{1}_{X>q}$, which introduces some moral hazard issues. This kind of reinsurance contract with indemnity $I(X)$ is thus not easy to sell in the traditional reinsurance marketplace. But, Moral hazard can be reduced by a coinsurance treaty (see for instance Cummins, Lalonde and Phillips (2004)). Moreover, the loss can also be written on a so-called loss index (see Cummins, Doherty and Lo (2002) for details). It avoids manipulation of the loss variable and indemnities can then be viewed as a portfolio of derivatives. For example, $I(X)$ is a long position on a call and a short position on a put and on a barrier bond. In this case, $(q-d)\mathbb{1}_{X>q}$ can be viewed as a barrier bond which is activated when the underlying loss X is above q. Our theoretical results, Propositions 1.1 and 2.1, show that it is optimal for the insurance company to sell the barrier

[14] Policies with upper limit on coverage could be derived from minimizing some risk measures under a mean variance premium principle. See Cummins and Mahul (2004).

bond corresponding to the right tail risk. Hence, $I(X)$ is attainable in an available capital market.

Conclusions

In this paper we derive the design of the optimal reinsurance contract to maximize the expected profit when the regulatory constraints are satisfied. We show that insurance companies have no incentives to protect themselves against extreme losses when regulatory requirements are based on Value-at-Risk or Conditional Tail Expectation. These results may partially confirm observed behaviors of insurance companies (Froot (2001)). Furthermore, we show that an alternative risk measures would lead insurance companies to fully hedge the right tail of the loss distribution.

The model in this paper is quite simple. There are no transaction cost for issuing and purchasing reinsurance contracts, no background risk, and a single loss during the period of insurance protection. Moreover both issuer and issued are risk neutral, both parties have symmetric (and perfect) information about the distribution of the loss. Even with the previously mentioned model limitations, the results of this paper could still be used as "prototypes" by insurance companies to design optimal risk management strategies, as well as by regulators to impose appropriate risk measures. Because of the similarities between the reinsurance market and the capital market, our results also present alternative risk transfers mechanisms in the capital market.

Proof of Proposition 1.1

Recall that final wealth W is given by $W = W_0 - P - X + I(X)$. Then, the event $\{W \geqslant W_0 - \nu\}$ is the same as $\{I(X) \geqslant P + X - \nu\}$ in terms of the coverage $I(X)$. The first step is to derive the optimal reinsurance coverage when the premium is fixed (Problem .1 below). In the second step Problem 1.1 is reduced to a sequence of Problem .1. As we have discussed in the main body of the text, the rationale of this approach follows from the property that $\mathbb{L}(P)$ is not monotone, consequently, the premium constraint is not necessary binding.

Problem .1 *Find the optimal reinsurance indemnity such that*

$$\min_{I(X)} \mathbb{P}\{W < W_0 - \nu\} \quad s.t. \quad \begin{cases} 0 \leqslant I(X) \leqslant X \\ \mathbb{E}\left[I(X) + C(I(X))\right] = P \end{cases}$$

Equivalently, Problem .1 is reformulated as follows.

$$\max_{I(X)} \mathbb{P}\{I(X) \geqslant P + X - \nu\} \;\; s.t. \;\; \begin{cases} 0 \leqslant I(X) \leqslant X \\ \mathbb{E}[I(X) + C(I(X))] = P \end{cases}$$

Lemma .1 *If Y^* satisfies the three following properties:*
(i) $0 \leqslant Y^* \leqslant X$,
(ii) $\mathbb{E}\left[Y^* + C\left(Y^*\right)\right] = P$,
(iii) There exists a positive $\lambda > 0$ such that for each $\omega \in \Omega$, $Y^*(\omega)$ is a solution of the following optimization problem:

$$\max_{Y \in [0, X(\omega)]} \left\{ \mathbb{1}_{P + X(\omega) - \nu \leqslant Y} - \lambda (Y + C(Y)) \right\}$$

then Y^* solves the optimization problem .1.

Proof. Given a coverage I which satisfies the constraints of the optimization problem .1. Therefore, using (iii), we have,

$$\forall \omega \in \Omega, \quad \mathbb{1}_{P + X(\omega) - \nu \leqslant Y^*} - \lambda (Y^* + C(Y^*)) \geqslant \mathbb{1}_{P + X(\omega) - \nu \leqslant I(\omega)} - \lambda (I(\omega) + C(I(\omega)))$$

Thus,

$$\mathbb{1}_{P + X(\omega) - \nu \leqslant Y^*(\omega)} - \mathbb{1}_{P + X(\omega) - \nu \leqslant I(\omega)} \geqslant \lambda \left(Y^*(\omega) + C(Y^*(\omega)) - I(\omega) - C(I(\omega)) \right).$$

We now take the expectation of this inequality. Therefore by condition (ii) one obtains,

$$\mathbb{P}\{P + X - \nu \leqslant Y^*\} - \mathbb{P}\{P + X - \nu \leqslant I\} \geqslant \lambda \left(P - \mathbb{E}\left[I + C(I)\right] \right)$$

Therefore, applying the constraints of the variable I, $\mathbb{E}[I(X) + C(I(X))] = P$,

$$\mathbb{P}\{P + X - Y^* \geqslant \nu\} - \mathbb{P}\{P + X - I \geqslant \nu\} \geqslant 0$$

The proof of this lemma is completed. $\square$

Lemma .2 *When $P \leqslant \nu$, each member of the following family $\{Y_\lambda\}_{\lambda>0}$ satisfies the conditions (i) and (iii) of Lemma .1.*

$$Y_\lambda(\omega) = \begin{cases} 0 & \text{if } X(\omega) < \nu - P \\ X(\omega) + P - \nu & \text{if } \nu - P \leqslant X(\omega) \leqslant \nu - P + D\left(\frac{1}{\lambda}\right) \\ 0 & \text{if } X(\omega) > \nu - P + D\left(\frac{1}{\lambda}\right) \end{cases}$$

where D is the inverse of $y \to y + C(y)$.

Proof. The property (i) is obviously satisfied. Indeed we only study the case when ν is more than the premium P.

First, if $X(\omega) < \nu - P$, then $P + X(\omega) - \nu < 0$, the function to maximize over $[0, X(\omega)]$ is equal to $1 - \lambda(Y + C(Y))$, decreasing over the interval $[0, X(\omega)]$ (since $C'(\cdot) > -1$), the maximum is thus obtained at $Y^*(\omega) = 0$.

Otherwise, $X(\omega) \geqslant \nu - P$. Since $P \leqslant \nu$, one has $\mathbb{P} + X(\omega) - \nu \leqslant X(\omega)$. We consider two cases: firstly, if $Y \in [0, P + X(\omega) - \nu)$, then the function to maximize is $-\lambda(Y + C(Y))$. It is decreasing with respect to the variable Y. Its maximum is 0, obtained at $Y = 0$. Secondly, if $Y \in [P + X(\omega) - \nu, X(\omega)]$, then the function to maximize is $1 - \lambda(Y + C(Y))$. It is decreasing. Its maximum is obtained at $Y = P + X(\omega) - \nu$ and its value is $1 - \lambda(P + X(\omega) - \nu + C(P + X(\omega) - \nu))$. We compare the value $1 - \lambda(P + X(\omega) - \nu + C(P + X(\omega) - \nu))$ and 0 to decide whether the maximum is attained at $Y = X(\omega) + P - \nu$ or $Y = 0$.

$$1 - \lambda(P + X(\omega) - \nu + C(P + X(\omega) - \nu)) \geqslant 0 \quad \Leftrightarrow \quad P + X(\omega) - \nu + C(P + X(\omega) - \nu) \leqslant \frac{1}{\lambda}.$$

Let $D = (Y + C(Y))^{-1}$ that exists since $Y + C(Y)$ is increasing, then:

$$X(\omega) \leqslant \nu - P + D\left(\frac{1}{\lambda}\right).$$

Lemma .2 is proved. $\square$

Proof of Proposition 1.1 Thanks to lemmas .1 and .2, it suffices to prove that there exists $\lambda > 0$ such that Y_λ defined in lemma .2 satisfies the condition (ii) of lemma .1. We then compute its associated cost function.

$$\mathcal{E}_\lambda := \mathbb{E}\left[(X + P - \nu)\,\mathbb{1}_{X \in \left[\nu - P, \nu - P + D\left(\frac{1}{\lambda}\right)\right]} + C\left((X + P - \nu)\,\mathbb{1}_{X \in \left[\nu - P, \nu - P + D\left(\frac{1}{\lambda}\right)\right]}\right)\right].$$

It is obvious then:

$$\lim_{\lambda \to 0^+} \mathcal{E}_\lambda = \mathbb{E}\left[(X - \nu + P)^+ + C\left((X - \nu + P)^+\right)\right], \qquad \lim_{\lambda \to +\infty} \mathcal{E}_\lambda = 0.$$

By Lebesgue dominance theorem we can easily prove the convergence property of $\mathcal{E}_\lambda$ with respect to the parameter λ. Then the existence of a solution $\lambda_P^* \in \mathbb{R}_+^*$ such that $\mathcal{E}_\lambda = P$ comes from the assumption on the continuous distribution of X and thus the continuity of $\mathcal{E}_\lambda$. Thus we have proved the first part of this Proposition. The second part follows easily from the first part. $\square$

Proposition 2.1

The solution of Problem 2.1 consists of two steps. We first solve Problem 2.1 by fixing the premium, reducing it to Problem .2 below.

Problem .2 *Solve the indemnity such that*

$$\min_{I(X)} \{\mathbb{E}[(W_0 - W)\mathbb{1}_{W_0 - W > \nu}]\} \quad s.t. \quad \begin{cases} 0 \leqslant I(X) \leqslant X \\ (1+\rho)\mathbb{E}[I(X)] = \Delta \end{cases}$$

Equivalently, Problem 2.1 is to minimize $\mathbb{E}[(P + X - I)\mathbb{1}_{P+X-I>\nu}]$ subject to the same constraints. Because of the linear premium principle, for the sake of simplicity we ignore the loading factor in the remainder proofs of Appendix A.

Lemma .3 *If Y^* satisfies the three following properties:*
(i) $0 \leqslant Y^* \leqslant X$,
(ii) $\mathbb{E}[Y^*] = \Delta$,
(iii) There exists a positive $\lambda > 1$ such that for each $\omega \in \Omega$, $Y^*(\omega)$ is a solution of the following optimization problem:

$$\min_{Y \in [0, X(\omega)]} \left\{(P + X(\omega) - Y)\,\mathbb{1}_{Y < P + X(\omega) - \nu} + \lambda Y\right\}$$

then Y^* solves Problem .2.

Proof. The proof of Lemma .3 is similar to the proof of Lemma .1. $\square$

Proof of Proposition 2.1: We use Lemma .3 and show that for $\lambda > 1$,

$$Y_\lambda(\omega) = \begin{cases} 0 & \text{if } X(\omega) < \nu - P \\ X(\omega) + P - \nu & \text{if } \nu - P \leqslant X(\omega) \leqslant \nu - P + \frac{\nu}{\lambda - 1} \\ 0 & \text{if } X(\omega) > \nu - P + \frac{\nu}{\lambda - 1} \end{cases}$$

satisfies conditions (i) and (iii) of Lemma .3. If $X(\omega) + P - \nu < 0$ then $Y^* = 0$. Otherwise $0 \leqslant P + X(\omega) - \nu < X$. Similar to the proof of Proposition 1.1 we can prove that $Y = P + X(\omega) - \nu$ is the maximum one if $\nu - P \leqslant X(\omega) \leqslant \nu - P + \frac{\nu}{\lambda - 1}$, else the maximum one is $Y = 0$ if $X(\omega) > \nu - P + \frac{v}{\lambda - 1}$.

Let

$$\mathcal{E}_\lambda := \mathbb{E}\left[(X + P - \nu)\,\mathbb{1}_{X \in \left(\nu - P, \nu - P + \frac{\nu}{\lambda - 1}\right)}\right].$$

It is obvious then:

$$\lim_{\lambda\to 1^+} \mathcal{E}_\lambda = \mathbb{E}\left[(X-\nu+P)^+\right], \qquad \lim_{\lambda\to+\infty} \mathcal{E}_\lambda = 0.$$

The existence of a solution $\lambda^* > 1$ such that $\mathcal{E}_\lambda = \Delta$ follows from the assumption on the continuous distribution of X and thus the continuity of $\mathcal{E}_\lambda$. Therefore, we have proved the first part of Proposition 2.1. The second part follows easily from the first part. $\square$

Proposition 2.2

Problem .3 *Find the optimal indemnity that solves:*

$$\min_{I(X)} \left\{ \; \mathbb{E}\left[(W_0 - W - v)^2 \mathbb{1}_{W_0 - W > \nu}\right] \; \right\} \quad s.t. \quad \begin{cases} 0 \leqslant I(X) \leqslant X \\ \mathbb{E}\left[I(X)\right] = \Delta \end{cases}$$

Lemma .4 *If Y^* satisfies the three following properties:*
(i) $0 \leqslant Y^* \leqslant X$,
(ii) $\mathbb{E}\left[Y^*\right] = \Delta$,
(iii) There exists a positive $\lambda > 0$ such that for each $\omega \in \Omega$, $Y^*(\omega)$ is a solution of the following optimization problem:

$$\min_{Y\in[0,X(\omega)]} \left\{(P + X(\omega) - Y - \nu)^2 \, \mathbb{1}_{Y<P+X(\omega)-\nu} + \lambda Y\right\}$$

then Y^* solves Problem .3.

Proof. Let Y^* be a random variable satisfying the three above conditions of the lemma. On the other hand, given another available payoff I which satisfies the constraints of the above optimization problem. Therefore, using (iii), we have, $\forall \omega \in \Omega$,

$$(P + X(\omega) - Y^*(\omega) - \nu)^2 \, \mathbb{1}_{Y^*(\omega)<P+X(\omega)-\nu} + \lambda Y^*(\omega) \leqslant \\ (P + X(\omega) - I(\omega) - \nu)^2 \, \mathbb{1}_{I(\omega)<P+X(\omega)-\nu} + \lambda I(\omega)$$

Thus,

$$(P + X(\omega) - Y^*(\omega) - \nu)^2 \, \mathbb{1}_{Y^*(\omega)<P+X(\omega)-\nu} - (P + X(\omega) - I(\omega) - \nu)^2 \, \mathbb{1}_{I(\omega)<P+X(\omega)-\nu} \\ \leqslant \lambda \left(I(\omega) - Y^*(\omega)\right)$$

We now take the expectation of the above inequality, therefore by condition (ii) one obtains,

$$\mathbb{E}\left[(P + X - Y^* - \nu)^2 \, \mathbb{1}_{Y^*<P+X-\nu}\right] - \mathbb{E}\left[(P + X - I - \nu)^2 \, \mathbb{1}_{I<P+X-\nu}\right] \leqslant \lambda \left(\mathbb{E}[I] - \Delta\right)$$

Therefore, applying the constraints of the variable I, $\mathbb{E}[I(X)] = \Delta$,

$$\mathbb{E}\left[(P + X - Y^* - \nu)^2 \, \mathbb{1}_{Y^*<P+X-\nu}\right] \leqslant \mathbb{E}\left[(P + X - I - \nu)^2 \, \mathbb{1}_{I<P+X-\nu}\right]$$

The proof of this lemma is completed. $\square$

Lemma .5 *When $P \leqslant \nu$, each member of the following family $\{Y_\lambda\}_{\lambda>0}$ satisfies the conditions (i) and (iii) of Lemma .4.*

$$Y_\lambda(\omega) = \begin{cases} 0 & \text{if } X(\omega) < \nu - P + \frac{\lambda}{2} \\ X(\omega) + P - \nu - \frac{\lambda}{2} & \text{if } \nu - P + \frac{\lambda}{2} \leqslant X(\omega) \end{cases}$$

Proof. The property (i) is obviously satisfied. We now prove the property (iii).

First, if $X(\omega) < \nu - P$, then $P + X(\omega) - v < 0$, the function to minimize over $[0, X(\omega)]$ is equal to λY, increasing over the interval $[0, X(\omega)]$, the minimum is thus obtained at $Y^*(\omega) = 0$.

Otherwise, $X(\omega) \geqslant \nu - P$. Since $P \leqslant \nu$, one has $P + X(\omega) - \nu \leqslant X(\omega)$. Thus we have to solve the optimization problem under the assumption $0 \leqslant P + X(\omega) - \nu \leqslant X(\omega)$. There are two cases: firstly, if $Y \in [0, P + X(\omega) - \nu)$, then the function to minimize is

$$\phi_1(Y) = (P + X(\omega) - \nu - Y)^2 + \lambda Y.$$

Its minimum is $\max\left(0, X(\omega) + P - \nu - \frac{\lambda}{2}\right)$. Secondly, if $Y \in [P + X(\omega) - \nu, X(\omega)]$, then the function to minimize is

$$\phi_2(Y) = \lambda Y.$$

Its minimum is obtained at $Y = P + X(\omega) - \nu$ and its value is $\lambda(P + X(\omega) - \nu)$. We then compare this value with the previous minimum:

- When $0 < X(\omega) + P - \nu - \frac{\lambda}{2}$, $\Phi_1\left(X(\omega) + P - \nu - \frac{\lambda}{2}\right) = \Phi_2(P + X(\omega) - \nu) - \frac{\lambda^2}{4} < \Phi_2(P + X(\omega) - \nu)$.
- When $0 > X(\omega) + P - \nu - \frac{\lambda}{2}$, $\Phi_1(0) = (P + X(\omega) - \nu)^2$. Since $\frac{\lambda}{2} > X(\omega) + P - \nu$, $\Phi_1(0) < \frac{\Phi_2(P+X(\omega)-\nu)}{2} < \Phi_2(P + X(\omega) - \nu)$.

Obviously, the minimum is thus obtained when $Y = \max\left(0, X(\omega) + P - \nu - \frac{\lambda}{2}\right)$. Lemma .5 is proved. $\square$

Proof of Proposition 2.2.

Thanks to both Lemmas .4 and .5, one only has to prove that there exists $\lambda > 0$ such that Y_λ defined in Lemma .5 satisfies the condition (ii) of Lemma .4. We then compute its expectation.

$$\mathcal{E}_\lambda := \mathbb{E}\left[\left(X + P - \nu - \frac{\lambda}{2}\right) \mathbb{1}_{X \in \left[\nu - P + \frac{\lambda}{2}, +\infty\right)}\right].$$

We see that

$$\lim_{\lambda \to 0^+} \mathcal{E}_\lambda = \mathbb{E}\left[(X - \nu + P)^+\right], \qquad \lim_{\lambda \to +\infty} \mathcal{E}_\lambda = 0.$$

The existence of a solution $\lambda^* \in \mathbb{R}_+^*$ such that $\mathcal{E}_\lambda = \Delta$ comes from the assumption on the continuous distribution of X and thus the continuity of $\mathcal{E}_\lambda$. Thus we have proved the first part of this Proposition. The second part follows easily from the first part. $\square$

References

Arrow, K. J. (1963), 'Uncertainty and the Welfare Economics of Medical Care', *The American Economic Review* **53**(5), 941–973.

Arrow, K. J. (1971), *Essays in the Theory of Risk Bearing*, Chicago: Markham.

Artzner, P., Delbaen, F., Eber, J.-M. & Heath, D. (1999), 'Coherent Measures of Risk', *Mathematical Finance* **9**, 203–228.

Basak, S. & Shapiro, A. (2001), 'Value-at-Risk-Based Risk Management: Optimal Policies and Asset Prices', *The Review of Financial Studies* **14**(2), 371–405.

Borch, K. (1971), 'Equilibrium in a Reinsurance Market', *Econometrica* **30**, 424–44.

Boyle, P. & Tian, W. (2007), 'Portfolio Management with Constraints', *Mathematical Finance* **17**(3), 319–344.

Caillaud, B., Dionne, G. & Jullien, B. (2000), 'Corporate Insurance with Optimal Financial Contracting', *Economic Theory* **16**, 77–105.

Cummins, J. D., Doherty, N. A. & Lo, A. (2002), 'Can Insurers Pay for the "Big One"? Measuring the Capacity of an Insurance Market to Respond to Catastrophic Losses', *Journal of Banking and Finance* **26**, 557–583.

Cummins, J. D., Lalonde, D. & Phillips, R. D. (2004), 'The Basis Risk of Catastrophic-Loss Index Securities', *Journal of Financial Economics* **71**, 77–111.

Cummins, J. D. & Mahul, O. (2004), 'The Demand for Insurance With an Upper Limit on Coverage', *Journal of Risk and Insurance* **71**(2), 253–264.

Davidson, W. N., Cross, M. L. & Thornton, J. H. (1992), 'Corporate Demand for Insurance: Some Epirical and Theoretical Results', *Journal of Financial Services Research* **6**, 61–72.

Doherty, N. A. & Schlesinger, H. (1983), 'Optimal Insurance in Incomplete Markets', *The Journal of Political Economy* **91**(6), 1045–1054.

Froot, K. A. (2001), 'The Market for Catastrophe Risk: a Clinical Examination', *Journal of Financial Economics* **60**, 529–571.

Froot, K. A., Scharfstein, D. S. & Stein, J. C. (1993), 'Risk management: Coordinating corporate investment and financing policies', *Journal of Finance* **48**, 1629–1658.

Froot, K. A. & Stein, J. C. (1998), 'Risk management, capital budgeting and capital structure policy for financial institutions: An integrated approach', *Journal of Financial Economics* **47**, 55–82.

Furman, E. & Landsman, Z. (2006), 'Tail variance premium with applications for elliptical portfolio of risks', *Astin Bulletin* **36**(2), 433–462.

Gajek, L. & Zagrodny, D. (2004), ‘Reinsurance Arrangements Maximizing Insurer’s Survival Probability’, *Journal of Risk and Insurance* **71**(3), 421–435.

Gollier, C., Koehl, P.-F. & Rochet, J.-C. (1997), ‘Risk-taking behavior with limited liability and risk aversion’, *Journal of Risk and Insurance* **64**(2), 341–370.

Gollier, C. & Schlesinger, H. (1996), ‘Arrow’s Theorem on the Optimality of Deductibles: A Stochastic Dominance Approach’, *Economic Theory* **7**, 359–363.

Hardy, M. R. (2003), *Investment Guarantees: Modelling and Risk Management for Equity-Linked Life Insurance*, Wiley.

Hau, A. (2006), ‘The Liquidity Demand for Corporate Property Insurance’, *Journal of Risk and Insurance* **73**(2), 261–278.

Hoyt, R. E. & Khang, H. (2000), ‘On the Demand for Corporate Property Insurance’, *Journal of Risk and Insurance* **67**(1), 91–107.

Huberman, G., Mayers, D. & Smith, C. (1983), ‘Optimal Insurance Policy Indemnity Schedules’, *Bell Journal of Economics* **14**(2), 415–426.

Inui, K. & Kijima, M. (2005), ‘On the Significance of Expected Shortfall as a Coherent Risk Measure’, *Journal of Banking and Finance* **29**, 853–864.

Kaluszka, M. & Okolewski, A. (2008), ‘An Extension of Arrow’s Result on Optimal Reinsurance Contract’, *Journal of Risk and Insurance* **75**(2), 275–288.

Leippold, M. & Trojani and P. Vanini, F. (2006), ‘Equilibrium impact of value-at-risk regulation’, *Journal of Economic Dynamics and Control* **30**, 1277–1313.

Mayers, D. & Smith, C. W. (1982), ‘On the Corporate Demand for Insurance’, *Journal of Business* **55**(2), 281–296.

Meyer, J. & Ormiston, M. B. (1999), ‘Analyzing the Demand for Deductible Insurance’, *Journal of Risk and Uncertainty* **18**, 223–230.

Raviv, A. (1979), ‘The Design of an Optimal Insurance Policy’, *The American Economic Review* **69**(1), 84–96.

Roy, A. (1952), ‘Safety first and the holding of assets’, *Econometrica* **20**(3), 431–445.

Schlesinger, H. (1981), ‘The Optimal Level of Deductibility in Insurance Contracts’, *Journal of Risk and Insurance* **48**, 465–481.

Wang, C. P., Shyu, D. & Huang, H. H. (2005), ‘Optimal insurance design under a value-at-risk framework’, *Geneva Risk and Insurance Review* **30**, 161–179.

Yamori, N. (1999), ‘An Empirical Investigation of the Japanese Corporate Demand for Insurance’, *Journal of Risk and Insurance* **66**(2), 239–252.