Insurance Market Effects of Risk Management Metrics

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Outline

- **I** Optimal Risk Sharing in the Insurance Market (standard theory of optimal insurance design)
- **II** Optimal Risk Sharing in the Presence of Regulators
- **III** Methodology & Results: Model & Economic Implications
- **IV** Research Directions

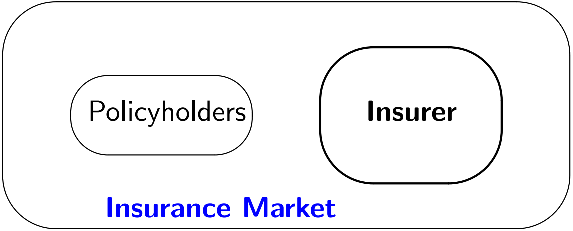
Part I

Optimal Risk Sharing

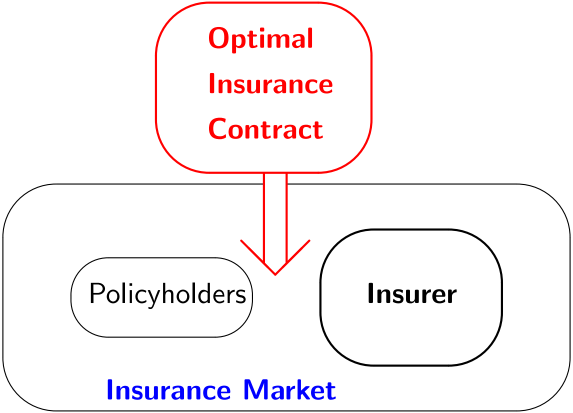
in the Insurance Market

(Standard theory of optimal insurance design)

Insurance Market Participants



Optimal Risk Sharing



The policyholder pays a premium P to the insurer. He has a loss X. And receives $I(X)$ from the insurance company.

Insurance Contract Design

Let $I(X)$ be an insurance indemnity.

$$\begin{cases} 0 \leqslant I(X) \leqslant X \\ P = \phi\left(E\left[I(X)\right]\right) \\ I(X) \ non-decreasing \end{cases}$$

with $\phi' > 0$ and $\phi(X) \geqslant X$.

Framework

- A one-period Model.
- At the **beginning** of the period:

$$\begin{aligned} W_0^p &: \text{Initial wealth of policyholders} \\ W_0 &: \text{Initial wealth of the insurer} \end{aligned}$$

- At the **end** of the period:

$$\begin{aligned} W_T^p &= W_0^p - P - X + I(X) \\ W_T &= W_0 + P - I(X) - c(I(X)) \end{aligned}$$

where X = Loss of policyholders, $c \geqslant 0$ and c is increasing.

- **U** : utility of policyholders, **V** : utility of the insurer.

Optimal Insurance Design

From the **policyholders**' perspective:

$$\max_I E\left[U(W_0^p - P - X + I(X))\right] \;\; s.t. \;\; \begin{cases} 0 \leqslant I(X) \leqslant X \\ P = \phi\left(E\left[I(X)\right]\right) \\ I(X) \;\; non - decreasing \end{cases}$$

From the **insurer**'s perspective:

$$\max_I E\left[V\left(W_0 + P - I(X) - c(I(X))\right)\right] \;\; s.t. \;\; \begin{cases} 0 \leqslant I(X) \leqslant X \\ P = \phi\left(E\left[I(X)\right]\right) \\ I(X) \;\; non - decreasing \end{cases}$$

Optimal Insurance Design from Policyholders' Perspective

From the **policyholders**' perspective:

$$\max_I \; E\left[U(W_0^p - P - X + I(X))\right] \;\; s.t. \;\; \begin{cases} 0 \leqslant I(X) \leqslant X \\ P = \phi\left(E\left[I(X)\right]\right) \\ I(X) \; non - decreasing \end{cases}$$

Stop loss insurance / Deductible are optimal (Arrow (1963)).

$$I^*(X) = \max(X - d, 0)$$

Uniqueness of the optimum a.s.

Optimal Insurance Design from the Insurer's Perspective

From the **insurer**'s perspective:

$$\max_{I} E\left[V(W_0 + P - I(X) - c(I(X)))\right] \;\; s.t. \; \begin{cases} 0 \leqslant I(X) \leqslant X \\ P = \phi\left(E\left[I(X)\right]\right) \\ I(X) \; non - decreasing \end{cases}$$

Upper-limit policies are optimal: (Raviv 1979)

$$I^*(X) = \min(X, c)$$

Uniqueness of the optimum a.s.

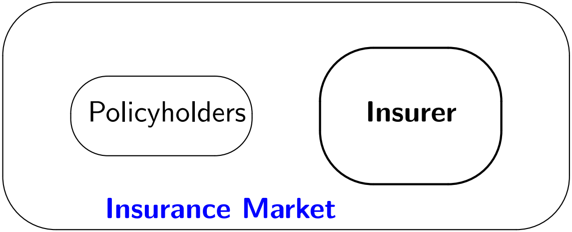
Part II

Optimal risk sharing

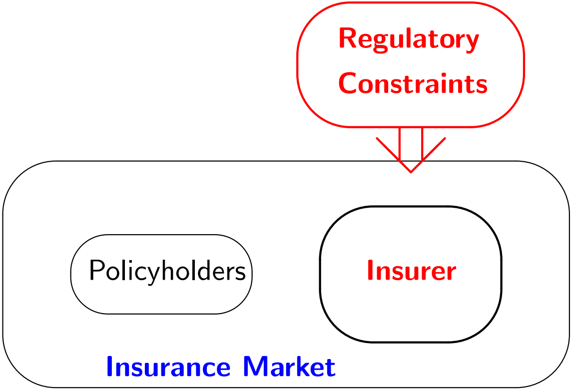
in the Presence of Regulators

(Value-at-Risk requirements)

Insurance Market Participants



Insurance Market Participants



Objective of our Study: the Insurance Market

In Europe, the **"Solvency II"** project will likely introduce **Value-at-Risk** requirements in the insurance marketplace.

What is the impact of such a change on the market?

We look at the **economic effects** of **Value-at-Risk** regulation imposed to **insurers** on **optimal risk sharing** in the **insurance** market.

Market **without** Regulators

Let $I(X)$ be an insurance indemnity.

$$\begin{cases} 0 \leqslant I(X) \leqslant X \\ \\ P = \phi\left(E\left[I(X)\right]\right) \\ \\ I(X)\ non - decreasing \end{cases}$$

with $\phi' > 0$ and $\phi(X) \geqslant X$.

$\Rightarrow$ Insurers can **sell any indemnity** I to customers (no constraints from regulators.)

Market **with** Regulators

Regulators aim at **protecting the insurance market** and **customers**. Thus, they want to induce companies to control their risks. Assume that regulators require companies to satisfy:

$$\mathrm{Pr}\,(\mathrm{W_T} \ < \ \mathrm{K}) \leqslant \alpha.$$

where W_T= insurer's final wealth.
$W_T = W_0 + P - I(X) - c(I(X))$ where c is non-negative and non-decreasing. The constraint writes also as:

$$\mathrm{Pr}\,(\mathrm{I(X)} \ > \ \mathrm{a}) \leqslant \alpha,$$

Obviously this condition can't always be satisfied.

- Ignoring the reinsurance market, the company is not fully free: some indemnities are TOO RISKY to be issued.

The presence of regulators influences the insurance market.

Part III

Methodology

& Results

Methodology

We compare two situations:

- Optimal Insurance Contracts **without** Regulation
- Optimal Insurance Contracts **under VaR Constraints**

We analyse optimal contracts:

- For *risk-averse policyholders* to buy,
- For *risk-averse insurers* to issue.

Market **without** Regulators

From the **policyholders**' perspective:

$$\max_I E\left[U(W_0^p - P - X + I(X))\right] \;\; s.t. \;\; \begin{cases} 0 \leqslant I(X) \leqslant X \\ P = \phi(E[I(X)]) \\ I(X) \; non - decreasing \end{cases}$$

From the **insurer**'s perspective:

$$\max_I E[V(W_0 + P - I(X) - c(I(X)))] \;\; s.t. \;\; \begin{cases} 0 \leqslant I(X) \leqslant X \\ P = \phi(E[I(X)]) \\ I(X) \; non - decreasing \end{cases}$$

Market **with** Regulators

From the **policyholders**' perspective:

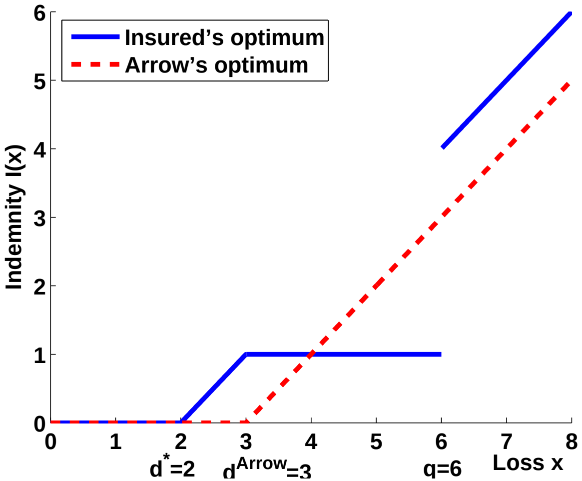
$$\max_{I} E\left[U(W_0^p - P - X + I(X))\right] \;\; s.t. \;\; \begin{cases} 0 \leqslant I(X) \leqslant X \\ P = \phi\left(E\left[I(X)\right]\right) \\ I(X) \;\; non - decreasing \\ \Pr\left(\mathrm{W_T} \;<\; \mathrm{K}\right) \leqslant \alpha \end{cases}$$

From the **insurer**'s perspective:

$$\max_{I} E\left[V\left(W_0 + P - I(X) - c(I(X))\right)\right] \;\; s.t. \;\; \begin{cases} 0 \leqslant I(X) \leqslant X \\ P = \phi\left(E\left[I(X)\right]\right) \\ I(X) \;\; non - decreasing \\ \Pr\left(\mathrm{W_T} \;<\; \mathrm{K}\right) \leqslant \alpha \end{cases}$$

Comparison of Optimal Insurance Designs for policyholders

In blue ——, with regulation. In **red** - - - , without regulation.



Comparison of Optimal Insurance Designs for Insurers

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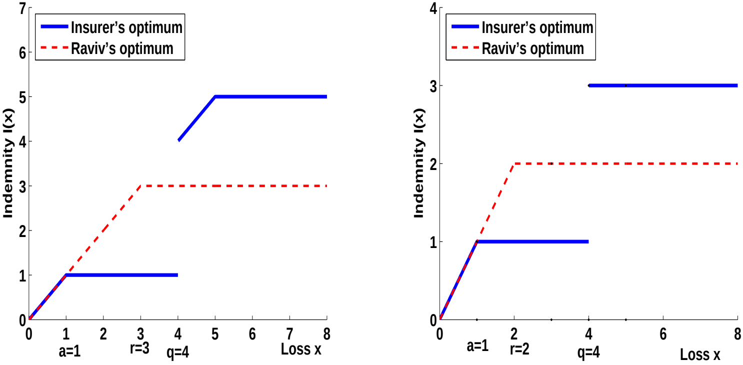

Contributions

- **Positive effects of VaR Regulation on the insurance market:**
  - Policyholders are *better* protected irrespective of the fact that the contract is designed from the insurer's perspective or policyholders' perspective.
  - Insurers' insolvency risk is reduced.

- Extend the work by Arrow (1963), Raviv (1979), Golubin (2006), Cummins and Mahul (2004).

- Technical contribution: Derive the optimal design under Value-at-Risk Constraints (non convex optimization with several interdependent constraints).

Negative Effects of VaR Regulation

Basak and Shapiro (RFS 2001) study the impact of Value-at-Risk risk management on the financial market. They derive the following economic effects:

- VaR risk managers incur larger losses when losses occur. **(Consistent with our results for the insurance market: it is optimal for VaR risk managers to incur losses in the worse states.)**
- Negative effects on the financial market:
  - Adverse effect of VaR regulation. Regulators should be concerned to reduce losses in any of the most adverse states of the world (and not to increase them).
  - VaR risk managers amplify stock market volatility at times of down market and attenuates the volatility in up market.
  (Optimal contracts are discontinuous under Value-at-Risk, creates moral hazard.)

Research Directions

- Towards the **equilibrium**: combine reinsurance market and insurance market ; design in a Pareto optimal framework.
- A **one-period framework**: how about multiperiod optimal contracts?
- Release assumptions:

  - **Premium** is based on the actuarial value

  - **Regulation** is based on Value-at-Risk

  - **Expected Utility Framework.**

- Optimal Insurance Design: Arrow (AER1963, book1971), Borch (Astin1962), Raviv (AER1979), Cummins and Mahul (JRI2004), Golubin (JRI2006), Doherty and Eeckhoudt (JRU 1995), Gollier and Schlesinger (ET1996), Eeckhoudt, Gollier and Schlesinger (Book2005), Gollier (2007 JPubEco), Doherty and Schlesinger (JPE2003), Huberman, Mayers and Smith (BellJoE1983).
- Risk Management for Financial Intermediation: Froot and Stein (JFE1998), Cummins, Phillips and Smith (2001); Cummins, Dionne, Gagné and Nouira (WP2007); Doherty and Dionne (JRU 1993); Froot, Scharfstein and Stein (JOF1993).
- Actuarial Approach: Gajek Zagrodny (IME2000, IME2004, JRI2004), Promislow and Young (IME 2005), Cai, Tan (Astin2007), Kaluszka (IME2001), Zhou Wu (IME2008).
- Finance literature: Basak and Shapiro (RFS2001), Shefrin and Statman (FM1993, JFQA), Kahneman and Tversky (E1979, JRU1992), Boyle and Tian (MF2007), Follmer and Leukert (F&S1999), Bernard, Boyle and Tian (WP2008).
- Mathematical Economics: Carlier and Dana (ET2003, JME2005), Picard (IER2000).