

# Optimal Investment with State-Dependent Constraints

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- ▶ This talk is joint work with **Phelim Boyle** (Wilfrid Laurier University, Waterloo, Canada) and with **Steven Vanduffel** (Vrije Universiteit Brussel (VUB), Belgium).

- ▶ Outline of the talk:

- 1 Characterization of optimal investment strategies for an investor with **law-invariant preferences** and a **fixed investment horizon**
- 2 Extension to the case when investors have **state-dependent constraints**.

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  - ② Extension to the case when investors have **state-dependent constraints**.

## **Part I:**

# **Optimal portfolio selection for law-invariant investors**

## Main Assumptions

- Consider an arbitrage-free market.
- Given a strategy with payoff  $X_T$  at time  $T$ . There exists  $Q$ , such that its price at 0 is

$$c(X_T) = \mathbb{E}_Q[e^{-rT} X_T]$$

- $P$  (“physical measure”) and  $Q$  (“risk-neutral measure”) are two equivalent probability measures:

$$\xi_T = e^{-rT} \left( \frac{dQ}{dP} \right)_T, \quad \mathbf{c}(\mathbf{X}_T) = \mathbb{E}_Q[e^{-rT} X_T] = \mathbb{E}_P[\xi_T \mathbf{X}_T].$$

We assume that all market participants agree on the state-price process  $\xi_T$ .

## Cost-efficient strategies

A strategy (or a payoff) is cost-efficient

if any other strategy that generates the same distribution under  $P$  costs at least as much.

### Simple Illustration

Example of

- $X_T \sim Y_T$  under  $P$
- but with different costs

in a 2-period binomial tree. ( $T = 2$ )

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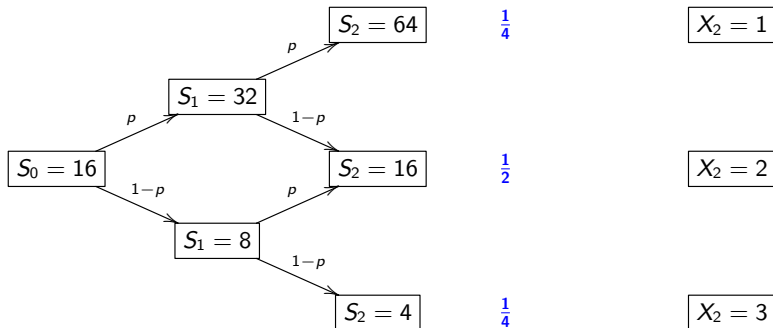
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## A simple illustration for $X_2$ , a payoff at $T = 2$

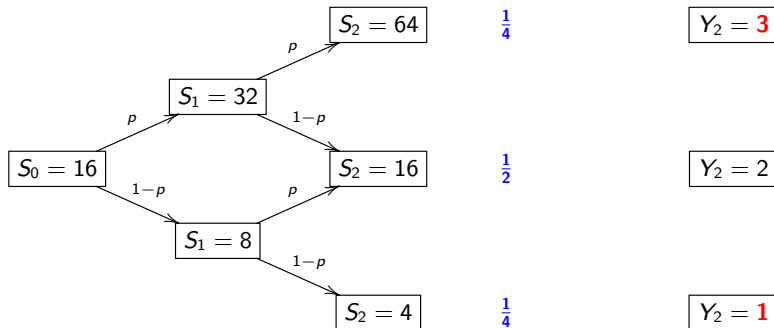
Real-world probabilities:  $p = \frac{1}{2}$





$Y_2$ , a payoff at  $T = 2$  distributed as  $X_2$

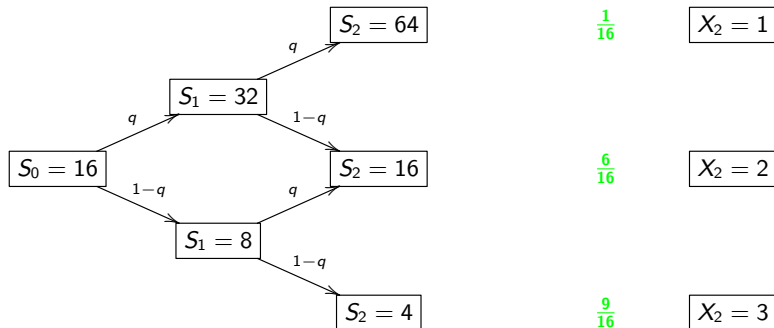
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$X_2$  and  $Y_2$  have the same distribution under the physical measure

$X_2$ , a payoff at  $T = 2$

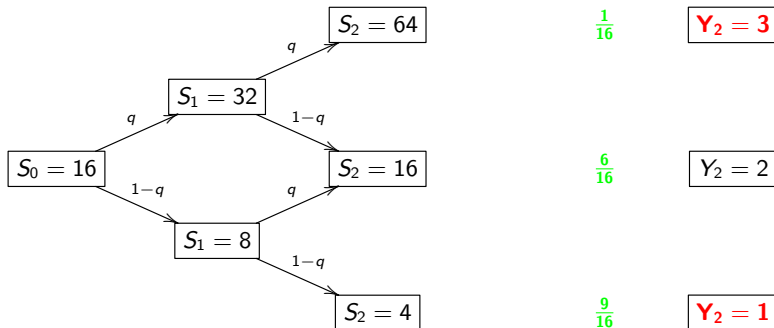
risk neutral probabilities:  $q = \frac{1}{4}$ .



$$c(X_2) = \text{Price of } X_2 = \left( \frac{1}{16} + \frac{6}{16}2 + \frac{9}{16}3 \right) = \frac{5}{2}$$

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## Traditional Approach to Portfolio Selection

Consider an investor with **increasing law-invariant** preferences and a **fixed** horizon. Denote by  $X_T$  the investor's final wealth.

- Optimize a law-invariant objective function
  - 1  $\max_{X_T} (E_P[U(X_T)])$  where  $U$  is increasing.
  - 2 Minimizing Value-at-Risk
  - 3 Probability target maximizing:  $\max_{X_T} P(X_T > K)$
  - 4 ...
- for a given **cost** (budget)

$$\text{cost at } 0 = E_Q[e^{-rT} X_T] = E_P[\xi_T X_T]$$

Find optimal strategy  $X_T^* \Rightarrow$  Optimal cdf  $F$  of  $X_T^*$

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## Our Approach

Consider an investor with

- Law-invariant preferences
- Increasing preferences
- A fixed investment horizon

The optimal strategy must be **cost-efficient**.

Therefore  $X_T^*$  in the previous slide is cost-efficient.

**Our approach:** We characterize cost-efficient strategies

(This characterization can then be used to solve optimal portfolio problems by restricting the set of possible strategies).

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## Sufficient Condition for Cost-efficiency

A subset  $A$  of  $\mathbb{R}^2$  is anti-monotonic if

for any  $(x_1, y_1)$  and  $(x_2, y_2) \in A$ ,  $(x_1 - x_2)(y_1 - y_2) \leq 0$ .

A random pair  $(X, Y)$  is anti-monotonic if

there exists an anti-monotonic set  $A$  of  $\mathbb{R}^2$  such that  $\mathbb{P}((X, Y) \in A) = 1$ .

Theorem (Sufficient condition for cost-efficiency)

*Any random payoff  $X_T$  with the property that  $(X_T, \xi_T)$  is **anti-monotonic** is **cost-efficient**.*

Note the absence of additional assumptions on  $\xi_T$  (it holds in discrete and continuous markets) and on  $X_T$  (no assumption on non-negativity).

## Idea of the proof

Minimizing the price  $c(X_T) = E[\xi_T X_T]$  when  $X_T \sim F$  amounts to finding the dependence structure that minimizes the correlation between the strategy and the state-price process

$$\begin{array}{ll} \min_{X_T} & \mathbb{E}[\xi_T X_T] \\ \text{subject to} & \begin{cases} X_T \sim F \\ \xi_T \sim G \end{cases} \end{array}$$

Recall that

$$\text{corr}(X_T, \xi_T) = \frac{\mathbb{E}[\xi_T X_T] - \mathbb{E}[\xi_T]\mathbb{E}[X_T]}{\text{std}(\xi_T)\text{std}(X_T)}.$$

We can prove that when the distributions for both  $X_T$  and  $\xi_T$  are fixed, we have

$(X_T, \xi_T)$  is anti-monotonic  $\Rightarrow \text{corr}[X_T, \xi_T]$  is minimal.

## Explicit Representation for Cost-efficiency

### Theorem

Consider the following optimization problem:

$$PD(F) = \min_{\{X_T \mid X_T \sim F\}} \mathbb{E}[\xi_T X_T]$$

**Assume  $\xi_T$  is continuously distributed**, then the optimal strategy is

$$X_T^* = F^{-1}(1 - F_\xi(\xi_T)).$$

Note that  $X_T^* \sim F$  and  $X_T^*$  is a.s. **unique** such that

$$PD(F) = c(X_T^*) = \mathbb{E}[\xi_T X_T^*]$$

## Idea of the proof (1/2)

Solving this problem amounts to finding bounds on copulas!

$$\begin{array}{ll} \min_{X_T} & \mathbb{E}[\xi_T X_T] \\ \text{subject to} & \begin{cases} X_T \sim F \\ \xi_T \sim G \end{cases} \end{array}$$

The distribution  $G$  is known and depends on the financial market.  
Let  $C$  denote a copula for  $(\xi_T, X)$ .

$$\mathbb{E}[\xi_T X] = \int \int (1 - G(\xi) - F(x) + C(G(\xi), F(x))) dx d\xi, \quad (1)$$

Bounds for  $\mathbb{E}[\xi_T X]$  are derived from bounds on  $C$

$$\max(u + v - 1, 0) \leq C(u, v) \leq \min(u, v)$$

(Fréchet-Hoeffding Bounds for copulas)

## Idea of the proof (2/2)

Consider a strategy with payoff  $X_T$  distributed as  $F$ . We define  $F^{-1}$  as follows:

$$F^{-1}(y) = \min \{x / F(x) \geq y\}.$$

Let  $Z = F_Z^{-1}(U)$ , then

$$E[F_Z^{-1}(U) F_X^{-1}(1 - U)] \leq E[F_Z^{-1}(U) X] \leq E[F_Z^{-1}(U) F_X^{-1}(U)]$$

In our setting, the cost of the strategy with payoff  $X_T$  is  $c(X_T) = E[\xi_T X_T]$ . Then, assuming that  $\xi_T$  is continuously distributed,

$$E[\xi_T F_X^{-1}(1 - F_\xi(\xi_T))] \leq c(X_T) \leq E[\xi_T F_X^{-1}(F_\xi(\xi_T))]$$

## Maximum price = Least efficient payoff

### Theorem

*Consider the following optimization problem:*

$$\max_{\{X_T \mid X_T \sim F\}} \mathbb{E}[\xi_T X_T]$$

**Assume  $\xi_T$  is continuously distributed.** *The unique strategy  $Z_T^*$  that generates the same distribution as  $F$  with the highest cost can be described as follows:*

$$Z_T^* = F^{-1}(F_\xi(\xi_T))$$

## Path-dependent payoffs are inefficient

### Corollary

*To be cost-efficient, the payoff of the derivative has to be of the following form:*

$$X_T^* = F^{-1}(1 - F_\xi(\xi_T))$$

*It becomes a European derivative written on  $S_T$  when the state-price process  $\xi_T$  can be expressed as a function of  $S_T$ . Thus path-dependent derivatives are in general not cost-efficient.*

### Corollary

*Consider a derivative with a payoff  $X_T$  which could be written as*

$$X_T = h(\xi_T)$$

*Then  $X_T$  is cost efficient if and only if  $h$  is non-increasing.*

## Black-Scholes Model

Under the physical measure  $P$ ,

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t^P$$

Then

$$\xi_T = e^{-rT} \left( \frac{dQ}{dP} \right) = a \left( \frac{S_T}{S_0} \right)^{-b}$$

where  $a = e^{\frac{\theta}{\sigma}(\mu - \frac{\sigma^2}{2})t - (r + \frac{\theta^2}{2})t}$  and  $b = \frac{\mu - r}{\sigma^2}$ .

**To be cost-efficient, the contract has to be a European derivative written on  $S_T$  and non-decreasing w.r.t.  $S_T$  (when  $\mu > r$ ). In this case,**

$$X_T^* = F^{-1}(F_{S_T}(S_T))$$



## Geometric Asian contract in Black-Scholes model

Assume a strike  $K$ . The payoff of the Geometric Asian call is given by

$$X_T = \left( e^{\frac{1}{T} \int_0^T \ln(S_t) dt} - K \right)^+$$

which corresponds in the discrete case to  $\left( \left( \prod_{k=1}^n S_{\frac{kT}{n}} \right)^{\frac{1}{n}} - K \right)^+$ .

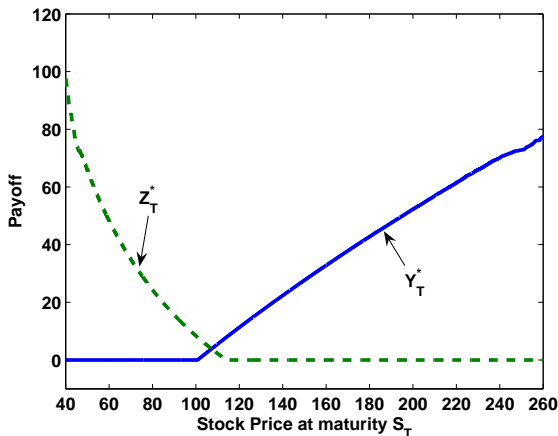
The efficient payoff that is distributed as the payoff  $X_T$  is a power call option

$$X_T^* = d \left( S_T^{1/\sqrt{3}} - \frac{K}{d} \right)^+$$

where  $d := S_0^{1-\frac{1}{\sqrt{3}}} e^{\left(\frac{1}{2}-\sqrt{\frac{1}{3}}\right)\left(\mu-\frac{\sigma^2}{2}\right)T}$ .

Similar result in the discrete case.

## Example: Discrete Geometric Option



With  $\sigma = 20\%$ ,  $\mu = 9\%$ ,  $r = 5\%$ ,  $S_0 = 100$ ,  $T = 1$  year,  $K = 100$ ,  $n = 12$ .

$$C(X_T^*) = 5.77 < \text{Price}(\text{geometric Asian}) = 5.94 < C(Z_T^*) = 9.03.$$

## Put option in Black-Scholes model

Assume a strike  $K$ . The payoff of the put is given by

$$L_T = (K - S_T)^+.$$

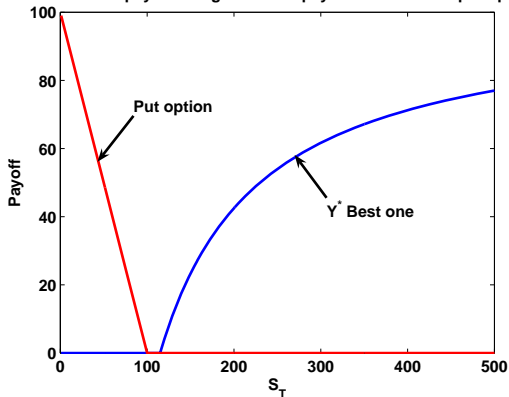
The payout that has the **lowest** cost and that has the same distribution as the put option payoff is given by

$$Y_T^* = F_L^{-1}(F_{S_T}(S_T)) = \left( K - \frac{S_0^2 e^{2\left(\mu - \frac{\sigma^2}{2}\right)T}}{S_T} \right)^+.$$

This type of power option “dominates” the put option.

## Cost-efficient payoff of a put

cost efficient payoff that gives same payoff distrib as the put option



With  $\sigma = 20\%$ ,  $\mu = 9\%$ ,  $r = 5\%$ ,  $S_0 = 100$ ,  $T = 1$  year,  $K = 100$ .

Distributional price of the put = 3.14

Price of the put = 5.57

Efficiency loss for the put =  $5.57 - 3.14 = 2.43$

## Explaining the Demand for Inefficient Payoffs

- ① **Other sources of uncertainty:** Stochastic interest rates or stochastic volatility
- ② **Transaction costs, frictions**
- ③ **Intermediary consumption.**
- ④ Often we are looking at an **isolated contract**: the theory applies to the complete portfolio.
- ⑤ **State-dependent needs**
  - **Background risk:**
    - Hedging a long position in the market index  $S_T$  (background risk) by purchasing a put option,
    - the background risk can be path-dependent.
  - **Stochastic benchmark or other constraints:** If the investor wants to outperform a given (stochastic) benchmark  $\Gamma$  such that:

$$P \{ \omega \in \Omega / W_T(\omega) > \Gamma(\omega) \} \geq \alpha.$$

## Part 2:

### Investment with State-Dependent Constraints

Problem considered so far

$$\min_{\{X_T \mid X_T \sim F\}} \mathbb{E} [\xi_T X_T].$$

A payoff that solves this problem is **cost-efficient**.

New Problem

$$\min_{\{Y_T \mid Y_T \sim F, \mathbb{S}\}} \mathbb{E} [\xi_T Y_T].$$

where  $\mathbb{S}$  denotes a set of constraints. A payoff that solves this problem is called a  **$\mathbb{S}$ -constrained cost-efficient payoff**.

## How to formulate “state-dependent constraints”?

$Y_T$  and  $S_T$  have given distributions.

- ▶ The investor wants to ensure a **minimum** when the market falls

$$\mathbb{P}(Y_T > 100 \mid S_T < 95) = 0.8.$$

This provides some additional information on the joint distribution between  $Y_T$  and  $S_T \Rightarrow$  information on the joint distribution of  $(\xi_T, Y_T)$  in the Black-Scholes framework.

- ▶  $Y_T$  is **decreasing** in  $S_T$  when the stock  $S_T$  falls below some level (to justify the demand of a put option).
- ▶  $Y_T$  is **independent** of  $S_T$  when  $S_T$  falls below some level.

All these constraints impose the strategy  $Y_T$  to pay out in given states of the world.

## Formally

**Goal:** Find the **cheapest** possible payoff  $Y_T$  with the distribution  $F$  and which **satisfies additional constraints** of the form

$$\mathbb{P}(\xi_T \leq x, Y_T \leq y) = Q(F_{\xi_T}(x), F(y)),$$

with  $x > 0, y \in \mathbb{R}$  and  $Q$  a given feasible function (for example a copula).

Each constraint gives information on the dependence between the state-price  $\xi_T$  and  $Y_T$  and is, for a given function  $Q$ , determined by the pair  $(F_{\xi_T}(x), F(y))$ .

**Denote the finite or infinite set of all such constraints by  $\mathbb{S}$ .**



## Sufficient condition for the existence

### Theorem

*Let  $t \in (0, T)$ . If there exists a copula  $L$  satisfying  $\mathbb{S}$  such that  $L \leq C$  (pointwise) for all other copulas  $C$  satisfying  $\mathbb{S}$  then the payoff  $Y_T^*$  given by*

$$Y_T^* = F^{-1}(f(\xi_T, \xi_t))$$

*is a  $\mathbb{S}$ -constrained cost-efficient payoff. Here  $f(\xi_T, \xi_t)$  is given by*

$$f(\xi_T, \xi_t) = \left( \ell_{F_{\xi_T}(\xi_T)} \right)^{-1} \left[ j_{F_{\xi_T}(\xi_T)}(F_{\xi_t}(\xi_t)) \right],$$

*where the functions  $j_u(v)$  and  $\ell_u(v)$  are defined as the first partial derivative for  $(u, v) \rightarrow J(u, v)$  and  $(u, v) \rightarrow L(u, v)$  respectively and where  $J$  denotes the copula for the random pair  $(\xi_T, \xi_t)$ .*

If  $(U, V)$  has a copula  $L$  then  $\ell_u(v) = \mathbb{P}(V \leq v | U = u)$ .

### Example 1: $\mathcal{S} = \emptyset$ (no constraints)

From the Fréchet-Hoeffding bounds on copulas one has

$$\forall (u, v) \in [0, 1]^2, \quad C(u, v) \geq \max(0, u + v - 1).$$

Note that  $L(u, v) := \max(0, u + v - 1)$  is a copula.

Then one obtains  $\ell_u(v) = 1$  if  $v > 1 - u$  and that  $\ell_u(v) = 0$  if  $v < 1 - u$ . Hence we find that  $\ell_u^{-1}(p) = 1 - u$  for all  $0 < p \leq 1$  which implies that

$$f(\xi_t, \xi_T) = 1 - F_{\xi_T}(\xi_T).$$

It follows that  $Y_T^*$  is given by

$$Y_T^* = F^{-1}(1 - (F_{\xi_T}(\xi_T)))$$

# Existence of the optimum $\Leftrightarrow$ Existence of minimum copula

## Theorem (Sufficient condition for existence of a minimal copula $L$ )

*Let  $\mathbb{S}$  be an increasing and compact subset of  $[0, 1]^2$ . Then a minimal copula  $L(u, v)$  satisfying  $\mathbb{S}$  exists and is given by*

$$L(u, v) = \max \{0, u + v - 1, K(u, v)\}.$$

*where  $K(u, v) = \max_{(a, b) \in \mathbb{S}} \{Q(a, b) - (a - u)^+ - (b - v)^+\}$ .*

Proof in Tankov (2011, Journal of Applied Probability).

**Consequently the existence of a  $\mathbb{S}$ –constrained cost-efficient payoff is guaranteed when  $\mathbb{S}$  is increasing and compact.**

## Theorem (Case of one constraint)

*Assume that there is only one constraint  $(a, b)$  in  $\mathbb{S}$  and let  $\vartheta := Q(a, b)$ . Then the minimum copula  $L$  is*

$$L(u, v) = \max \{0, u + v - 1, \vartheta - (a - u)^+ - (b - v)^+\}.$$

*The  $\mathbb{S}$ -constrained cost-efficient payoff  $Y_T^*$  exists and is unique. It can be expressed as*

$$Y_T^* = F^{-1}(G(F_{\xi_T}(\xi_T))), \quad (2)$$

*where  $G : [0, 1] \rightarrow [0, 1]$  is defined as  $G(u) = \ell_u^{-1}(1)$  and can be written as*

$$G(u) = \begin{cases} 1 - u & \text{if } 0 \leq u \leq a - \vartheta, \\ a + b - \vartheta - u & \text{if } a - \vartheta < u \leq a, \\ 1 + \vartheta - u & \text{if } a < u \leq 1 + \vartheta - b, \\ 1 - u & \text{if } 1 + \vartheta - b < u \leq 1. \end{cases} \quad (3)$$

## Example 2: \$ contains 1 constraint

Assume a Black-Scholes market. We suppose that the investor is looking for the payoff  $Y_T$  such that  $Y_T \sim F$  (where  $F$  is the cdf of  $S_T$ ) and satisfies the following constraint

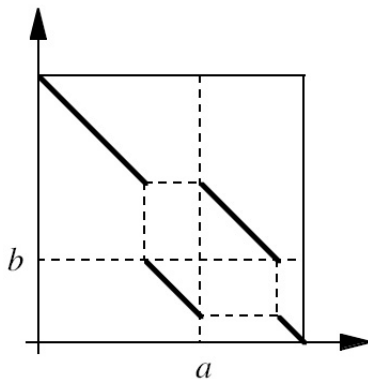
$$\mathbb{P}(S_T < 95, Y_T > 100) = 0.2.$$

The optimal strategy, where  $a = 1 - F_{S_T}(95)$ ,  $b = F_{S_T}(100)$  and  $\vartheta = 0.2 - F_{S_T}(95) + F_{S_T}(100)$  is given by the previous theorem.

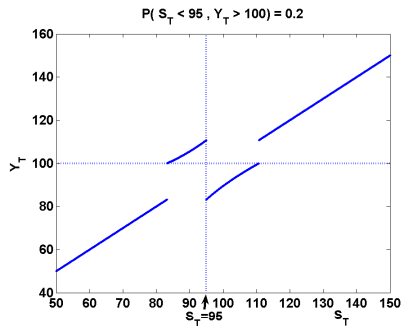
Its price is 100.2

## Example 2: Illustration

Minimum Copula



Optimal Strategy



### Example 3: $\mathbb{S}$ is infinite

A cost-efficient strategy with the same distribution  $F$  as  $S_T$  but such that it is decreasing in  $S_T$  when  $S_T \leq \ell$  is unique a.s. Its payoff is equal to

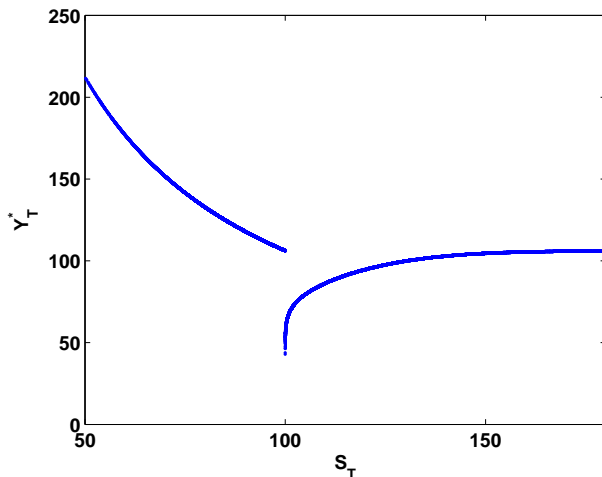
$$Y_T^* = F^{-1} [G(F(S_T))],$$

where  $G : [0, 1] \rightarrow [0, 1]$  is given by

$$G(u) = \begin{cases} 1 - u & \text{if } 0 \leq u \leq F(\ell), \\ u - F(\ell) & \text{if } F(\ell) < u \leq 1. \end{cases}$$

The **constrained cost-efficient payoff** can be written as

$$Y_T^* := F^{-1} [(1 - F(S_T)) \mathbb{1}_{S_T < \ell} + (F(S_T) - F(\ell)) \mathbb{1}_{S_T \geq \ell}].$$



$Y_T^*$  as a function of  $S_T$ . Parameters:  $\ell = 100$ ,  $S_0 = 100$ ,  $\mu = 0.05$ ,  $\sigma = 0.2$ ,  $T = 1$  and  $r = 0.03$ . The price is 103.4.

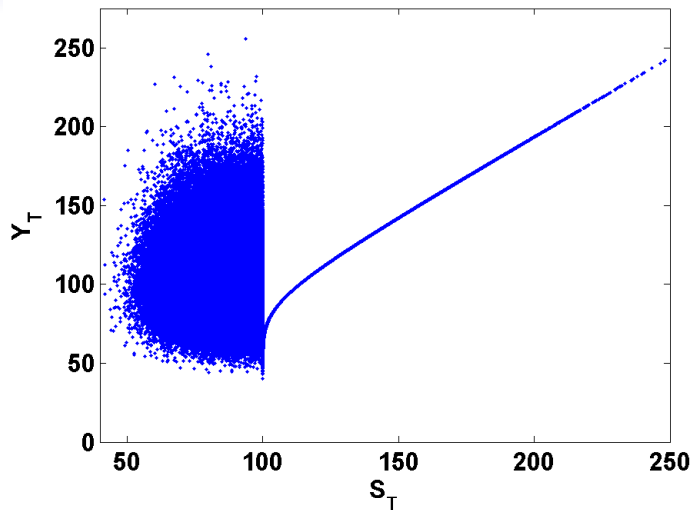


### Example 4: $\mathbb{S}$ is infinite

A cost-efficient strategy with the same distribution  $F$  as  $S_T$  but such that it is independent of  $S_T$  when  $S_T \leq \ell$  can be constructed as

$$Y_T^* = F^{-1} \left( \Phi(k(S_t, S_T)) \mathbb{1}_{S_T < \ell} + \left( \frac{F(S_T) - F(\ell)}{1 - F(\ell)} \right) \mathbb{1}_{S_T \geq \ell} \right),$$

where  $k(S_t, S_T) = \frac{\ln\left(\frac{S_t}{S_T^{t/T}}\right) - (1 - \frac{t}{T}) \ln(S_0)}{\sigma \sqrt{t - \frac{t^2}{T}}}$  and  $t \in (0, T)$  can be chosen freely (**No uniqueness and path-independence anymore**).



10,000 realizations of  $Y_T^*$  as a function of  $S_T$  where  $\ell = 100$ ,  $S_0 = 100$ ,  $\mu = 0.05$ ,  $\sigma = 0.2$ ,  $T = 1$ ,  $r = 0.03$  and  $t = T/2$ . Its price is 101.1

## Conclusion

- Cost-efficiency: a preference-free framework for ranking different investment strategies.
- Characterization of cost-efficient strategies.
- For a given investment strategy, we derive an explicit analytical expression for the cheapest and the most expensive strategies that have the same payoff distribution.
- Optimal investment choice under state-dependent constraints.  
**In the presence of state-dependent constraints, optimal strategies**
  - are not always non-decreasing with the stock price  $S_T$ .
  - are not anymore unique and could be path-dependent.

## Further Research Directions / Work in Progress

- ▶ Using cost-efficiency to derive **bounds for insurance prices** derived from indifference utility pricing (working paper on “Bounds for Insurance Prices” with Steven Vanduffel)
- ▶ Extension to the presence of **stochastic interest rates** and application to executive compensation (work in progress with Jit Seng Chen and Phelim Boyle).
- ▶ Further extend the work on state-dependent constraints:
  - ① Solve with **expectations constraints** between  $\xi_T$  and  $X_T$ .

$$\mathbb{E}[g_i(\xi_T, X_T)] \in I_i$$

where  $I_i$  is an interval, possibly reduced to a single value.

- ② Solve with the probability constraint of outperforming a benchmark

$$\mathbb{P}(X_T > h(S_T)) \geq \varepsilon$$

- ③ Extend the literature on optimal portfolio selection in specific models under state-dependent constraints.

*Do not hesitate to contact me to get updated working papers!*

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