Structured Investment Products with Caps and Floors

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Outline

- **I** The Retail Structured Products Market.
  Example: locally-capped globally-floored contracts.
- **II** Why do retail investors buy locally-capped contract? A puzzle
- **III** Evidence from the market
- **IV** Complexity of locally-capped contracts.
- **V** Overweighting high returns and impact on decision making.

What is a structured product?

- **A structured product is an investment vehicle that provides a particular payoff related to some reference portfolio (Index, security, stock, basket).**
- Structured products are sold by financial institutions such as **banks** and **insurance companies** (variable annuities, equity indexed annuities)
- They have become very **popular**.
  - Volume of exchange listed structured products is about **$50 billion** for the period 1992-2005 in US.
  - Volume of Equity Indexed Annuities sold in the US in 2004 alone is estimated to **$25 billion**.
  - Annual Variable annuities sales in USA is currently about **$200 billion**.

Different variations

Structured product design can be modified and extended in countless ways.

- Guaranteed floor
- Upper limits (local cap, global cap)
- Path-dependent payoff (Asian, lookback, barrier)
- Multi-period based payments: **locally-capped contracts**

Example of a locally-capped contract

- AMEX Ticker: JPL.E
- Issuer: JP Morgan Chase
- Underlying: S&P500
- Maturity: 5 years
- Initial investment: \$1,000
- Payoff$= \max(\$1,100 \;\; ; \;\; \$1,000 + \textit{additional amount})$
- In the prospectus dated June 22, 2004:
  "The additional amount will be calculated by the calculation agent by multiplying \$1,000 by the sum of the quarterly capped Index returns for each of the 20 quarterly valuation periods during the term of the notes."

Payoff of a locally-capped globally-floored contract

- Initial investment= \$1,000
- Maturity $T = 5$ years
- Let $g = 10\%$ be the minimum guaranteed rate at maturity.
- X_T: Locally-capped design (Quarterly Local Cap $c = 6\%$).

$$X_T = 1,000 + 1,000 \max\left(10\% \ , \ \sum_{i=1}^{20} \min\left(6\%, \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}} \right) \right)$$

- The contract consists of:
  - a zero coupon bond with maturity amount $\$1,100$.
  - a complex option component
- It is often overpriced but popular.

Local Cap vs Global Cap

- Initial investment= \$1
- Maturity $T = 5$ years
- Let $g = 10\%$ be the minimum guaranteed rate after 5 years.
- Y_T: GC design (Global Cap C)

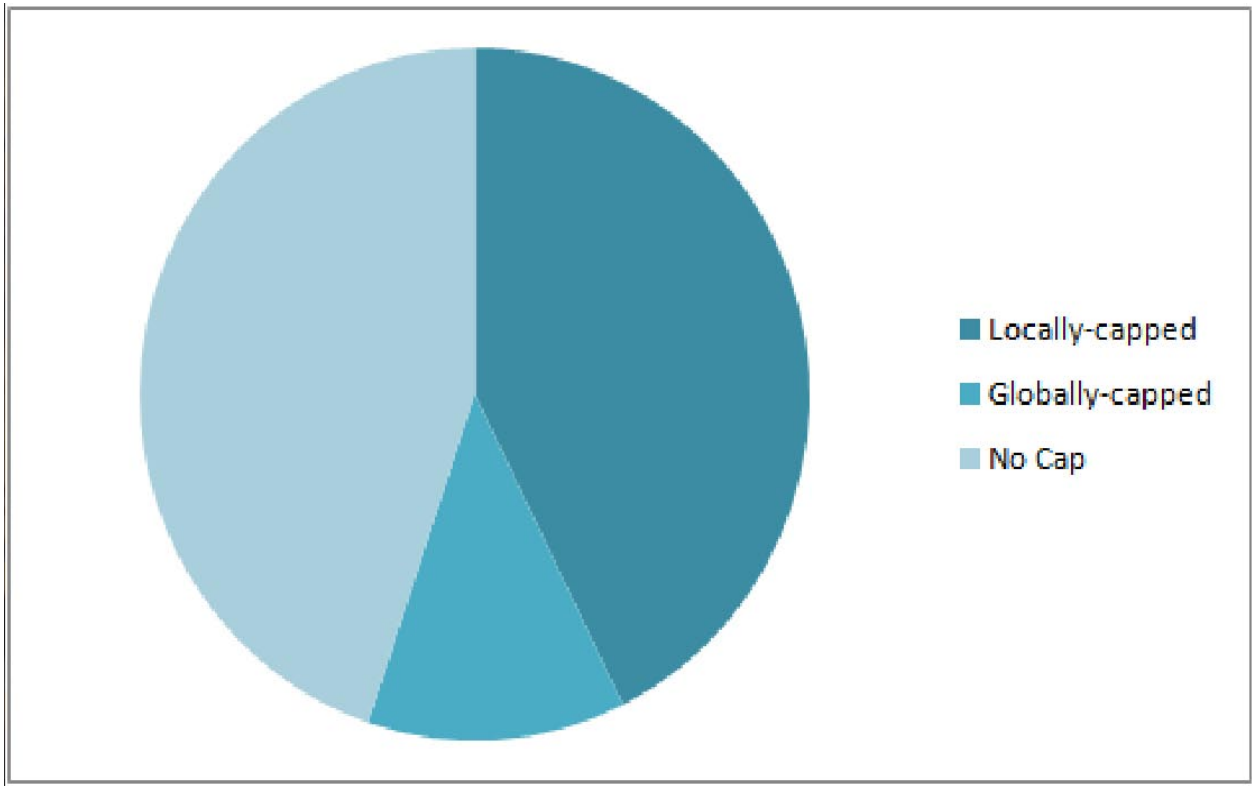
$$Y_T = 1 + \max\left(g \ , \ \min\left(C, \frac{S_T - S_0}{S_0} \right) \right)$$

(long position in a bond and in a standard call option and short position in another standard call option.)

- X_T: LC design (Local Cap c on the quarterly returns).

$$X_T = 1 + \max\left(g \ , \ \sum_{i=1}^{20} \min\left(c, \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}} \right) \right)$$

locally-capped globally-floored contracts
Volume in the Exchange-listed Index Linked Notes (May 2008)



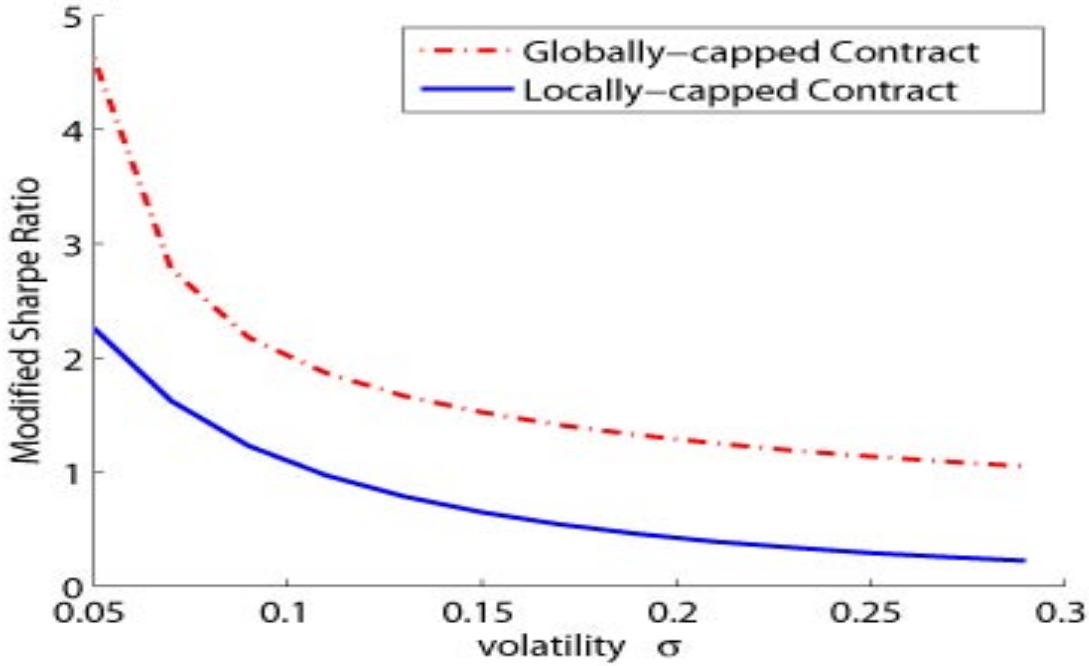
Mean Variance Investors

- Let Z_0 be the initial investment
- Let the guarantee be $(1+g)Z_0$ at the maturity T.
- We define the **modified Sharpe ratio** as follows

$$R_Z = \frac{\mathrm{E}[Z_T] - Z_0(1+g)}{\mathrm{std}(Z_T)}$$

- We compute this ratio for the quarterly-capped contract R_X and for the globally-capped contract R_Y.

Mean Variance Investors



- The Quarterly Sum cap has a quarterly cap of 8.7%, a global floor $g = 10\%$ and a maturity $T = 5$ years.
- For each volatility, the global cap is such that the GC contract has the same no-arbitrage price as the 8.7% quarterly-capped (which is equal to 920\$).
- Other parameters $r = 5\%$, $\delta = 2\%$, $\mu = 0.09$.

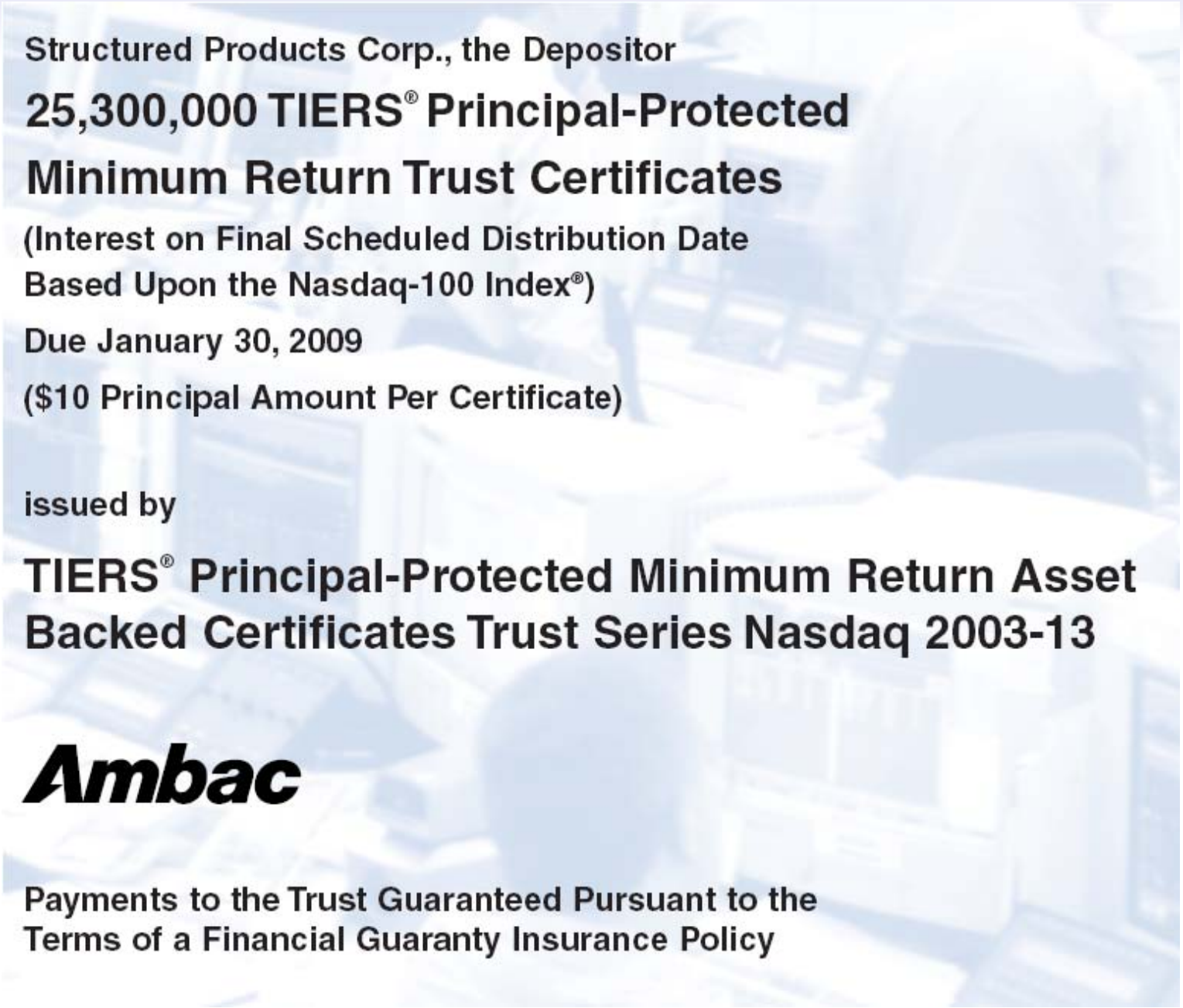
Summary

- **Mean variance investors** ought to prefer the globally capped contract to the locally capped contract.
- We also did some further experiments with risk-averse investors (with an exponential utility for instance) and show that there are two key factors that explain the investor's preferences for the locally-capped contracts:
  1. **the volatility:**
     - **When volatility is high**, risk averse investors often prefer the globally capped contract to the locally capped contract.
     - **If volatility is low**, locally-capped contracts can be of interest to moderate risk averse investors.
  2. **the risk aversion.** Very-risk averse investors prefer the globally-capped contracts for any volatility.

Possible Explanations

- Retail investors are convinced by sales agents to buy it because they have high commissions.
- Investors may be influenced by the bias in the hypothetical projections displayed in the prospectuses to **overweight** the probabilities of receiving the maximum possible return.
- The **complexity** of the contract confuses investors and they make inappropriate choices (Carlin (2006)).

Overweighting Evidence


Structured Products Corp., the Depositor
25,300,000 TIERS® Principal-Protected
Minimum Return Trust Certificates
(Interest on Final Scheduled Distribution Date
Based Upon the Nasdaq-100 Index®)
Due January 30, 2009
($10 Principal Amount Per Certificate)
issued by
TIERS® Principal-Protected Minimum Return Asset
Backed Certificates Trust Series Nasdaq 2003-13
Ambac
Payments to the Trust Guaranteed Pursuant to the
Terms of a Financial Guaranty Insurance Policy

Characteristic of this locally-capped contract

- AMEX Ticker: NAS
- Based on the *NAS: Nasdaq-100 Index*.
- The initial investment is \$10
- The maturity payoff is a compounded monthly-capped returns
- Capped at 5.5% per month.
- In the prospectus, there is a description of 7 hypothetical examples.

Example 1: The value of the Nasdaq-100 Index as of the final scheduled distribution date is greater than its value at issuance and the Nasdaq-100 Index appreciated by 3.00% (an amount less than the periodic appreciation cap) during each period throughout the term of the certificates:

	2003		2004		2005		2006		2007		2008		2009	
	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return
January			1,515	3.00%	2,160	3.00%	3,079	3.00%	4,390	3.00%	6,259	3.00%	8,924	3.00%
February			1,560	3.00%	2,224	3.00%	3,171	3.00%	4,522	3.00%	6,447	3.00%		
March			1,607	3.00%	2,291	3.00%	3,267	3.00%	4,657	3.00%	6,640	3.00%		
April..........			1,655	3.00%	2,360	3.00%	3,365	3.00%	4,797	3.00%	6,839	3.00%		
May			1,705	3.00%	2,431	3.00%	3,465	3.00%	4,941	3.00%	7,045	3.00%		
June			1,756	3.00%	2,504	3.00%	3,569	3.00%	5,089	3.00%	7,256	3.00%		
July			1,809	3.00%	2,579	3.00%	3,677	3.00%	5,242	3.00%	7,474	3.00%		
August	1,307	3.00%	1,863	3.00%	2,656	3.00%	3,787	3.00%	5,399	3.00%	7,698	3.00%		
September	1,346	3.00%	1,919	3.00%	2,736	3.00%	3,900	3.00%	5,561	3.00%	7,929	3.00%		
October	1,386	3.00%	1,976	3.00%	2,818	3.00%	4,017	3.00%	5,728	3.00%	8,167	3.00%		
November	1,428	3.00%	2,036	3.00%	2,902	3.00%	4,138	3.00%	5,900	3.00%	8,412	3.00%		
December	1,471	3.00%	2,097	3.00%	2,989	3.00%	4,262	3.00%	6,077	3.00%	8,664	3.00%		

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&(1.00+0.03)\,]\ \text{minus } 1.00 = 6.0349 \text{ or } 603.49\%.
\end{aligned}
$$

Interest distribution amount $= \$10.00 \times 6.0349 = \$60.35$

Payment on the final scheduled distribution date $= \$10.00 + \$60.35 = \$70.35$ per certificate.

Example 2: The value of the Nasdaq-100 Index as of the final scheduled distribution date is greater than its value at issuance and the Nasdaq-100 Index appreciated by 5.50% (an amount equal to the periodic appreciation cap) during each period throughout the term of the certificates:

	2003		2004		2005		2006		2007		2008		2009	
	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return
January			1,749	5.50%	3,325	5.50%	6,322	5.50%	12,020	5.50%	22,852	5.50%	43,447	5.50%
February			1,845	5.50%	3,508	5.50%	6,670	5.50%	12,681	5.50%	24,109	5.50%		
March			1,947	5.50%	3,701	5.50%	7,037	5.50%	13,378	5.50%	25,435	5.50%		
April			2,054	5.50%	3,905	5.50%	7,424	5.50%	14,114	5.50%	26,834	5.50%		
May			2,167	5.50%	4,120	5.50%	7,832	5.50%	14,891	5.50%	28,310	5.50%		
June			2,286	5.50%	4,346	5.50%	8,263	5.50%	15,710	5.50%	29,867	5.50%		
July			2,412	5.50%	4,585	5.50%	8,717	5.50%	16,574	5.50%	31,510	5.50%		
August	1,338	5.50%	2,544	5.50%	4,837	5.50%	9,197	5.50%	17,485	5.50%	33,243	5.50%		
September	1,412	5.50%	2,684	5.50%	5,103	5.50%	9,703	5.50%	18,447	5.50%	35,071	5.50%		
October	1,490	5.50%	2,832	5.50%	5,384	5.50%	10,236	5.50%	19,461	5.50%	37,000	5.50%		
November	1,571	5.50%	2,988	5.50%	5,680	5.50%	10,799	5.50%	20,532	5.50%	39,035	5.50%		
December	1,658	5.50%	3,152	5.50%	5,993	5.50%	11,393	5.50%	21,661	5.50%	41,182	5.50%		

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\begin{aligned}
\text{Index return} = [\,&(1.00 + 0.055) \times (1.00 + 0.055) \times (1.00 + 0.055) \times (1.00 + 0.055) \times (1.00 + 0.055) \\
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&0.055)\,] \text{ minus } 1.00 = 33.2501 \text{ or } 3{,}325.01\%.
\end{aligned}
$$

This is the maximum possible index return.

Interest distribution amount $= \$10.00 \times 33.2501 = \$332.50$

Because the periodic capped return for any reset period will not in any circumstances be greater than 5.50%, $332.50 is the maximum possible interest distribution amount.

Example 3: The value of the Nasdaq-100 Index as of the final scheduled distribution date is greater than its value at issuance and the Nasdaq-100 Index appreciated by 7.00% (an amount greater than the periodic appreciation cap) during each period throughout the term of the certificates:

	2003		2004		2005		2006		2007		2008		2009	
	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return
January			1,904	5.50%	4,288	5.50%	9,656	5.50%	21,748	5.50%	48,980	5.50%	110,313	5.50%
February ...			2,037	5.50%	4,588	5.50%	10,332	5.50%	23,270	5.50%	52,409	5.50%		
March			2,180	5.50%	4,909	5.50%	11,055	5.50%	24,899	5.50%	56,078	5.50%		
April			2,332	5.50%	5,252	5.50%	11,829	5.50%	26,642	5.50%	60,003	5.50%		
May			2,495	5.50%	5,620	5.50%	12,657	5.50%	28,507	5.50%	64,203	5.50%		
June			2,670	5.50%	6,013	5.50%	13,543	5.50%	30,502	5.50%	68,697	5.50%		
July			2,857	5.50%	6,434	5.50%	14,491	5.50%	32,638	5.50%	73,506	5.50%		
August	1,357	5.50%	3,057	5.50%	6,885	5.50%	15,506	5.50%	34,922	5.50%	78,652	5.50%		
September ..	1,452	5.50%	3,271	5.50%	7,367	5.50%	16,591	5.50%	37,367	5.50%	84,157	5.50%		
October	1,554	5.50%	3,500	5.50%	7,882	5.50%	17,753	5.50%	39,983	5.50%	90,048	5.50%		
November ..	1,663	5.50%	3,745	5.50%	8,434	5.50%	18,995	5.50%	42,781	5.50%	96,352	5.50%		
December ..	1,779	5.50%	4,007	5.50%	9,025	5.50%	20,325	5.50%	45,776	5.50%	103,096	5.50%		

*Actual return on the Nasdaq-100 Index during each reset period is 7.00%, but because of the 5.50% cap the periodic capped return would be 5.50%.

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\begin{gathered}
\text{Index return} = [(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\ (1.00+0.055)\times(1.00+0.055)\times \\
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(1.00+0.055)]\ \text{minus}\ 1.00 = 33.2501\ \text{or}\ 3{,}325.01\%.
\end{gathered}
$$

This is the maximum possible index return.

Interest distribution amount = $10.00 × 33.2501 = $332.50

Because the periodic capped return for any reset period will not in any circumstances be greater than 5.50%, $332.50 is the maximum possible interest distribution amount.

Payment on the final scheduled distribution date = $10.00 + $332.50 = $342.50 per certificate.

This is the maximum possible payment on the final scheduled distribution date.

Example 4: ***The value of the Nasdaq-100 Index as of the final scheduled distribution date is less than its value at issuance and the Nasdaq-100 Index declined steadily throughout the term of the certificates:***

	2003		2004		2005		2006		2007		2008		2009	
	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return
January			1,082	−1.30%	971	−1.90%	741	−2.50%	525	−3.10%	346	−3.70%	211	−4.30%
February ...			1,166	−1.35%	952	−1.95%	722	−2.55%	509	−3.15%	333	−3.75%		
March			1,149	−1.40%	933	−2.00%	703	−2.60%	493	−3.20%	320	−3.80%		
April			1,133	−1.45%	914	−2.05%	685	−2.65%	477	−3.25%	308	−3.85%		
May			1,116	−1.50%	894	−2.10%	666	−2.70%	461	−3.30%	296	−3.90%		
June			1,098	−1.55%	875	−2.15%	648	−2.75%	445	−3.35%	284	−3.95%		
July			1,081	−1.60%	856	−2.20%	630	−2.80%	430	−3.40%	273	−4.00%		
August	1,255	−1.05%	1,063	−1.65%	837	−2.25%	612	−2.85%	415	−3.45%	262	−4.05%		
September ..	1,241	−1.10%	1,045	−1.70%	817	−2.30%	594	−2.90%	401	−3.50%	251	−4.10%		
October	1,227	−1.15%	1,027	−1.75%	798	−2.35%	577	−2.95%	387	−3.55%	241	−4.15%		
November ..	1,212	−1.20%	1,008	−1.80%	779	−2.40%	559	−3.00%	373	−3.60%	231	−4.20%		
December ..	1,197	−1.25%	990	−1.85%	760	−2.45%	542	−3.05%	359	−3.65%	221	−4.25%		

Index return = [(1.00 + −0.0105) × (1.00 + −0.0110) × (1.00 + −0.0115) × (1.00 + −0.0120) × (1.00 + −0.0125) × (1.00 + −0.0130) × (1.00 + −0.0135) × (1.00 + −0.0140) × (1.00 + −0.0145) × (1.00 + −0.0150) × (1.00 + −0.0155) × (1.00 + −0.0160) × (1.00 + −0.0165) × (1.00 + −0.0170) × (1.00 + −0.0175) × (1.00 + −0.0180) × (1.00 + −0.0185) × (1.00 + −0.0190) × (1.00 + −0.0195) × (1.00 + −0.0200) × (1.00 + −0.0205) × (1.00 + −0.0210) × (1.00 + −0.0215) × (1.00 + −0.0220) × (1.00 + −0.0225) × (1.00 + −0.0230) × (1.00 + −0.0235) × (1.00 + −0.0240) × (1.00 + −0.0245) × (1.00 + −0.0250) × (1.00 + −0.0255) × (1.00 + −0.0260) × (1.00 + −0.0265) × (1.00 + −0.0270) × (1.00 + −0.0275) × (1.00 + −0.0280) × (1.00 + −0.0285) × (1.00 + −0.0290) × (1.00 + −0.0295) × (1.00 + −0.0300) × (1.00 + −0.0305) × (1.00 + −0.0310) × (1.00 + −0.0315) × (1.00 + −0.0320) × (1.00 + −0.0325) × (1.00 + −0.0330) × (1.00 + −0.0335) × (1.00 + −0.0340) × (1.00 + −0.0345) × (1.00 + −0.0350) × (1.00 + −0.0355) × (1.00 + −0.0360) × (1.00 + −0.0365) × (1.00 + −0.0370) × (1.00 + −0.0375) × (1.00 + −0.0380) × (1.00 + −0.0385) × (1.00 + −0.0390) × (1.00 + −0.0395) × (1.00 + −0.0400) × (1.00 + −0.0405) × (1.00 + −0.0410) × (1.00 + −0.0415) × (1.00 + −0.0420) × (1.00 + −0.0425) × (1.00 + −0.0430)] minus 1.00 = −0.8335 or −83.35% but the index return cannot be less than 7.00%

Interest distribution amount = $10.00 × 0.07 = $0.70

Payment on the final scheduled distribution date = $10.00 + $0.70 = $10.70 per certificate, the amount of your original investment plus the minimum return of 7.00%.

Example 5: ***The value of the Nasdaq-100 Index as of the final scheduled distribution date is greater than its value at issuance and the Nasdaq-100 Index increased steadily throughout all but one of the reset periods during the term of the certificates. If the decline is greater than or equal to approximately 96.71% for one reset period, the index return will not be greater than the minimum return of 7.00%.***

	2003		2004		2005		2006		2007		2008		2009	
	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return	Index Level	Capped Return
January			1,749	5.50%	3,325	5.50%	6,322	5.50%	12,020	5.50%	713	5.50%	1,355	5.50%
February			1,845	5.50%	3,508	5.50%	6,670	5.50%	12,681	5.50%	752	5.50%		
March			1,947	5.50%	3,701	5.50%	7,037	5.50%	13,378	5.50%	793	5.50%		
April			2,054	5.50%	3,905	5.50%	7,424	5.50%	14,114	5.50%	837	5.50%		
May			2,167	5.50%	4,120	5.50%	7,832	5.50%	14,891	5.50%	883	5.50%		
June			2,286	5.50%	4,346	5.50%	8,263	5.50%	15,710	5.50%	931	5.50%		
July			2,412	5.50%	4,585	5.50%	8,717	5.50%	517	-96.71%	983	5.50%		
August	1,338	5.50%	2,544	5.50%	4,837	5.50%	9,197	5.50%	545	5.50%	1,037	5.50%		
September	1,412	5.50%	2,684	5.50%	5,103	5.50%	9,703	5.50%	575	5.50%	1,094	5.50%		
October	1,490	5.50%	2,832	5.50%	5,384	5.50%	10,236	5.50%	607	5.50%	1,154	5.50%		
November	1,571	5.50%	2,988	5.50%	5,680	5.50%	10,799	5.50%	640	5.50%	1,217	5.50%		
December	1,658	5.50%	3,152	5.50%	5,993	5.50%	11,393	5.50%	675	5.50%	1,284	5.50%		

Index return $[(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times$
$(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+$
$0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times$
$(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+$
$0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times$
$(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+$
$0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times$
$(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+$
$0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+-0.967)\times(1.00+0.055)\times$
$(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+$
$0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times$
$(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+0.055)\times(1.00+$
$0.055)]$ minus $1.00 = 0.0681$ or 6.81% but the index return cannot be less than 7.00%

Interest distribution amount = $\$10.00 \times 0.07 = \$0.70$

Payment on the final scheduled distribution date = $\$10.00 + \$0.70 = \$10.70$ per certificate, the amount of your original investment plus the minimum return of 7.00% (even though the value of the Nasdaq-100 Index increased in all but one of the reset periods).

Observations

- Most outrageous set of unrealistic assumptions we observed.
- In the 3 first examples, the final payoffs are respectively $1.03^{66} = \$60.35$, $1.055^{66} = \$332.5$, $1.055^{66} = \$332.5$.
- Empirical probability of a monthly return exceeding 5.5% is 0.2 (1971-2008).
- Assuming an i.i.d. distribution of the monthly returns, the probability of the maximum possible return is

$$0.2^{66} = 7 \times 10^{-47}$$

  which is an impossible event.
- Getting returns such as in Examples 4 and 5 have an historical probability of about 50% of taking place.
- these securities are also subject to default risk.

Overview

- Our analysis of the hypothetical examples presented in the 39 prospectuses reveals that the above description is common practice.
- All issuers provide in their prospectus 4 to 7 hypothetical examples. One or two of the first three examples assumes that the investor receives the maximum possible return.
- We suggest that including these illustrations as hypothetical scenarios provides very concrete evidence of attempts to overweight the probabilities of obtaining the maximum possible return.

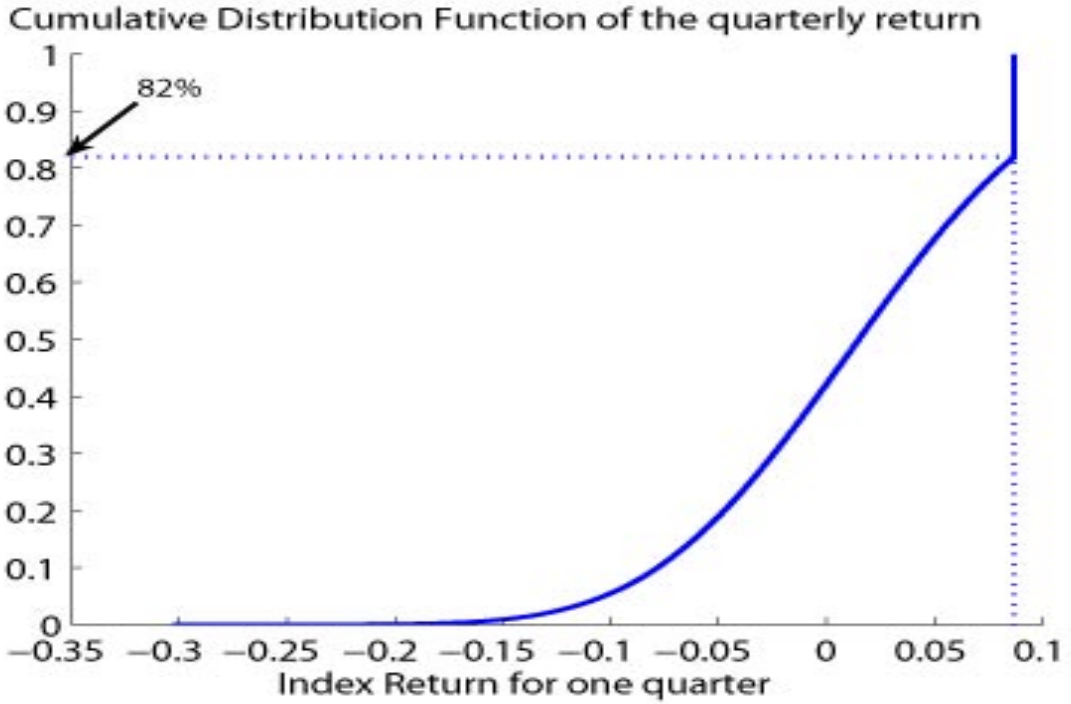
Complexity Evidence

Distribution of the Payoff of a Quarterly Sum Cap

1. The distribution of the payoff of a Quarterly Sum Cap is extremely difficult for investors to have a realistic representation of the sum of periodically capped returns.
2. The reason stems from how the cap affects the final distribution of returns.

Distribution of a Monthly return capped at 8.7%

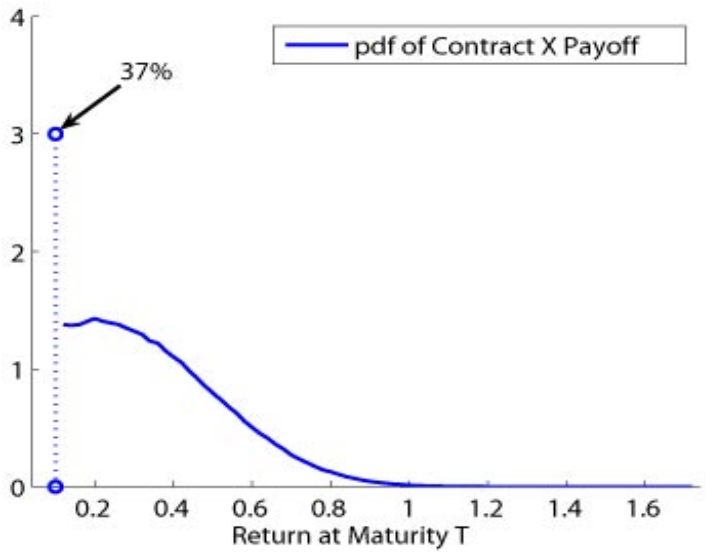
Because of the presence of a cap the return the quarterly-capped return has a truncated distribution function as shown

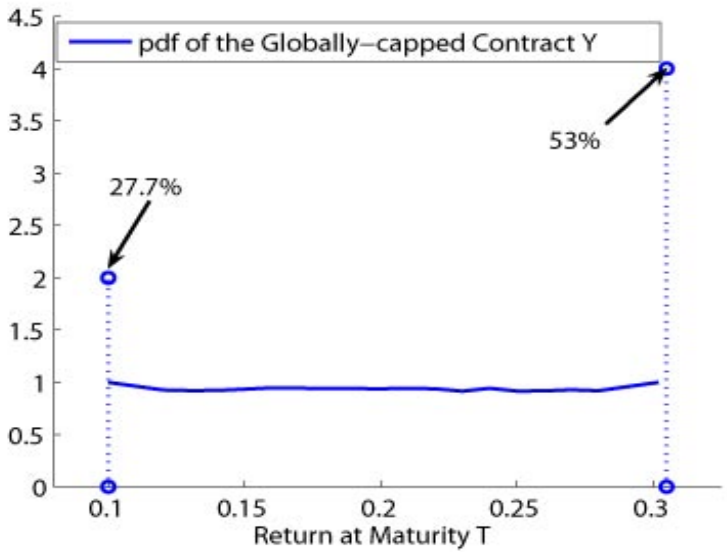


- If R denotes the quarterly return, the graph is $\Pr(R \leqslant x)$.
- A probability mass of 18% at the cap level
- Parameters are set to $r = 5\%$, $\delta = 2\%$, $\mu = 0.09$, $\sigma = 15\%$ (benchmark economic assumptions).

Comparison Local Cap and Global Cap

- Minimum guaranteed rate of 10% (global floor) over T years.
- The left panel is the density of the payoff under the Quarterly Sum Cap (X). The right panel corresponds to the density of the payoff under the globally-capped contract (Y).
- Parameters are set to $r = 5\%$, $\delta = 2\%$, $\mu = 0.09$, $\sigma = 15\%$.





Effects of Complexity

A locally-capped contract is complicated:

- sales agents can draw attention to the maximum attainable return
- Distribution of the payoff is not intuitive

This is consistent with Carlin (2006) model.

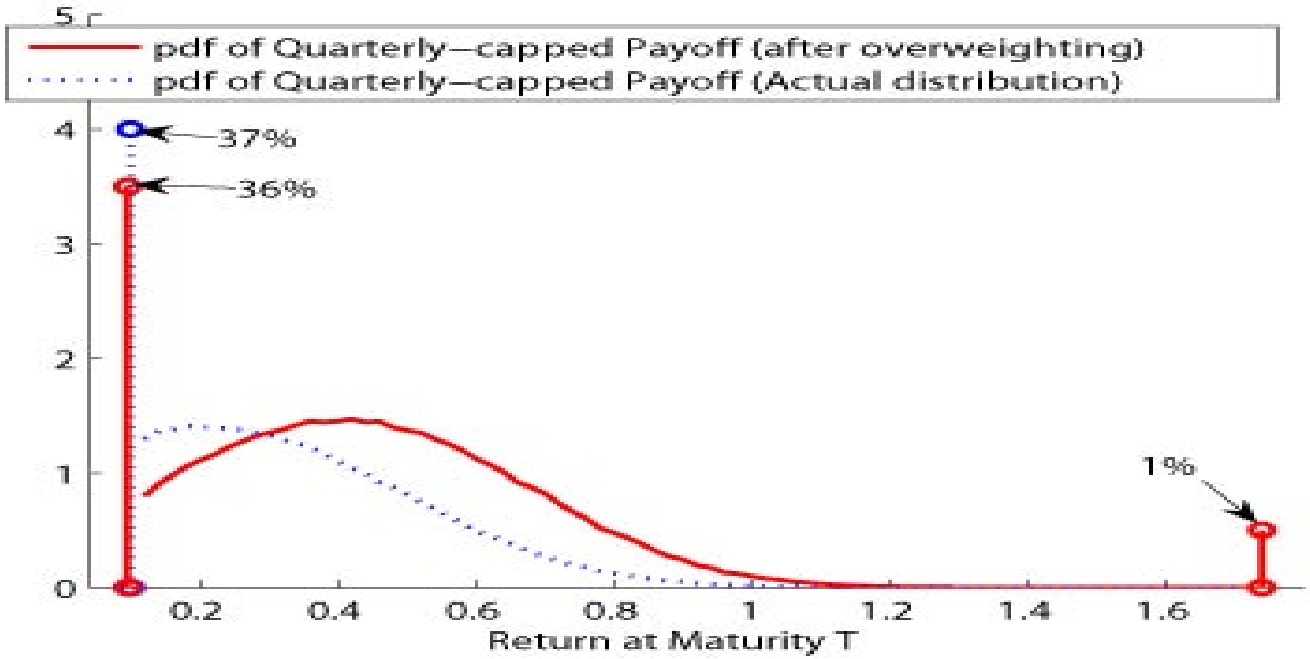
- sellers of retail financial products deliberately design them to be complicated in order to confuse consumers and increase profits.
- producers will increase the complexity of their financial products in order to overprice them.
- customers choose randomly.

Overweighting Technique

1. increase the drift of the underlying index
2. add a lump of probability at the extreme right end of the distribution.

Density of the payoff under the Quarterly Sum Cap (X) with an additional expected annual Index return of 5%.

The quarterly cap is $c = 8.7\%$, $r = 5\%$, $\mu = 9\%$, $\delta = 2\%$, $\sigma = 15\%$.



Impact on Decision Making

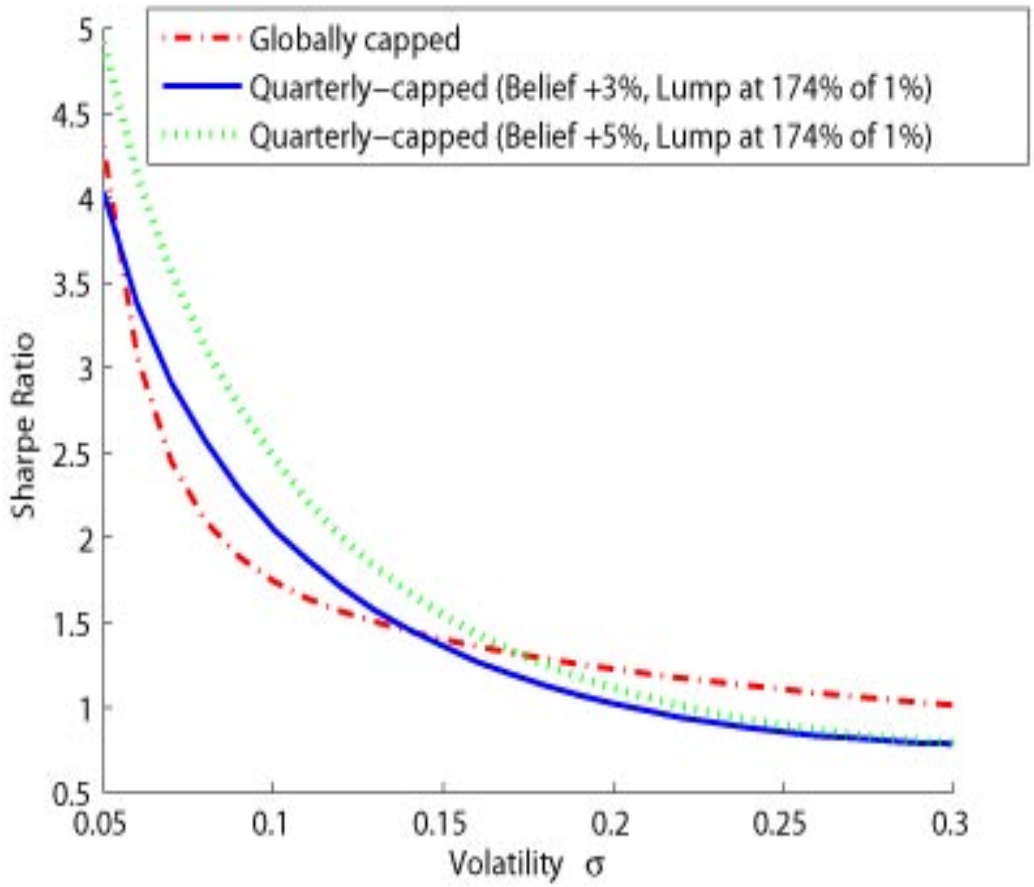
- Modified Sharpe ratio using the new measure for the quarterly Sum Cap and the original measure for the other contract:

$$\tilde{R}_X = \frac{\mathrm{E}_Q[Z_T] - Z_0(1+g)}{\mathrm{std}_Q(Z_T)}$$

- Compare of $\tilde{R}_X$ with R_Y
- 8.7% quarterly cap, $g = 10\%$, $T = 5$ years.
- Other parameters $r = 5\%$, $\delta = 2\%$, $\mu = 0.09$.

Impact on Decision Making

The quarterly-capped contract has a 8.7% quarterly cap, $g = 10\%$, $T = 5$ years. For each volatility, the cap of the globally-capped contract is such that the contract has the same no-arbitrage price as the 8.7% quarterly-capped contract. Investors overweight the tail of the distributions. Other parameters $r = 5\%$, $\delta = 2\%$, $\mu = 0.09$.



Impact on Decision Making

- Mean variance investors may prefer the locally-capped contract if they sufficiently overweight the probability of getting the maximum possible return.
- The relative attractiveness of the locally capped contract declines as the assumed volatility increases.
- Both of these effects are also observed in the case of more general utility functions.

Conclusions

- We describe some popular design in the market: locally-capped contracts.
- The demand for these complex products is puzzling.
- We provide a possible explanation based on investor misperception of the return distribution where low probability events of high returns are overweighted.
- We provide evidence that this tendency is encouraged by the hypothetical examples in the prospectus supplements.
- The demand for these products might be similar to the demand for premium bonds.