Natural Balance Sheet Hedge of Equity Indexed Annuities

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Introduction

Equity Linked Insurance Market

- Contracts sold by **insurance companies** (Variable Annuities, Equity Indexed Annuities, Unit-linked contracts...)
- They usually provide a **complicated payoff related to some reference portfolio**. The payoff design can be modified and extended in countless ways. Here are some of them:
  - Guaranteed **floor** (periodically or at maturity)
  - Upper limits or **caps**
  - Path-dependent payoffs (Asian, lookback, barrier), **locally-capped contracts** and **cliquet options**
  - Embedded complex life benefits: GM**X**B
- They have become very **popular** in many countries (the total VA assets in the US were $1.41 trillion as of June 30, 2008.)

Current Economic Context:

- **New regulation and new accounting standards** (proposed by the IASB (International Accounting Standards Board) in Europe and by the FASB (Financial Accounting Standards Board) in the US.
- **"fair value"** or **"mark-to-market"** reporting system: Insurers are required to evaluate EIAs at their market value in their balance sheet
- Europe, US, Australia and Asia are adopting or about to adopt such systems.

However such change in the regulation is highly **controversial**...

Controversial Change

See for instance Jørgensen (2004), Ballotta, Haberman and Wang (2005), Plantin, Sapra and Shin (2004).

- **positive** because
  - "the market value of a liability is more relevant than historical cost... it reflects the amount at which that liability could be incurred or settled in a current transaction between willing parties."
  - More transparency.
- **negative** because
  - "market values" cannot be obtained if there exists no actual liquid market.
  - market values increase the volatility of the annual results of companies and is contrary to the smooth return policyholders and shareholders would prefer.
  - reporting standards might induce excessive volatility in the markets.

Many Interesting Issues about EIAs

- **Pricing, hedging and risk management**. Market values.
- **Design** from buyers' perspective *(choice of the right (optimal) contract to buy)*.
- **Design** from insurers' perspective *(choice of the right portfolio of policies to sell).*
  - We show **how to stabilize aggregate liabilities market value by building a portfolio of policies.**
  - Insurers can immunize their balance sheet against market changes and parameter uncertainty by carefully combining different payoffs.

Outline of the paper

- Description of common contracts
- Natural Hedge of volatility risk.
- Effects of embedded ratchet options or annual guarantee.

Two popular designs

Initial investment= \$M

We focus on two popular designs sold by insurance companies:

- **Standard Equity Indexed Annuities** (participating policy) with payoff given by:

$$X_T = M \max\left(e^{gT}, k\frac{S_T}{S_0}\right)$$

  where k is called the participating rate and g stands for the minimum guaranteed rate at maturity.

- Periodically-capped contracts. Ex: **Monthly Sum Cap** with cap level equal to c on the return of each month.

Monthly Sum Cap

- Initial investment= \$M
- Minimum guaranteed rate g at maturity T years.
- Local Cap c on the monthly return.
- Let $t_0 = 0$, $t_1 = \frac{1}{12}, t_2 = \frac{2}{12}, ..., t_n = \frac{n}{12} = T$. The payoff Z_T of the monthly sum cap is linked to

$$\sum_{i=1}^{n} \min\left(c, \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}} \right)$$

Monthly Sum Cap (c =3%), T =1 year, Year 2003.

Month	Raw S&P return	Adjusted Return used for Monthly Sum Cap
1	-2.74	-2.74
2	-1.70	-1.70
3	0.84	0.84
4	**8.10**	**3.00**
5	**5.09**	**3.00**
6	1.13	1.13
7	1.62	1.62
8	1.79	1.79
9	-1.19	-1.19
10	**5.50**	**3.00**
11	0.71	0.71
12	**5.07**	**3.00**

The sum of the adjusted returns in the third column is 12.45%.

Monthly Sum Cap (c =3%), T =1 year, Year 2008.

Month	Raw S&P return	Adjusted Return used for Monthly Sum Cap
1	-6.12	-6.12
2	-3.48	-3.48
3	-0.60	-0.60
4	4.75	**3.00**
5	1.07	1.07
6	-8.60	-8.60
7	-0.99	-0.99
8	1.22	1.22
9	-9.08	-9.08
10	-16.94	-16.94
11	-7.48	-7.48
12	0.78	0.78

The sum of the adjusted returns in the third column is **-47.2%**.

Monthly Sum Cap Contract

- Initial investment= \$M
- Minimum guaranteed rate g at maturity T years.
- Local Cap c on the monthly return.
- Let $t_0 = 0$, $t_1 = \frac{1}{12}, t_2 = \frac{2}{12}, ..., t_n = \frac{n}{12} = T$. The payoff Z_T of the monthly sum cap contract is

$$Z_T = M \max\left(e^{gT} , 1 + \sum_{i=1}^{n} \min\left(c, \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}} \right) \right)$$

- The contract consists of:
  - a zero-coupon bond
  - a complex option component

Pricing by Monte Carlo or by Fast Fourier analysis.

Natural Hedge for Insurers

What is a "natural hedge"?

Well-known example, to hedge mortality risk, life insurance companies can offer simultaneously two types of policies to people in the same age class:

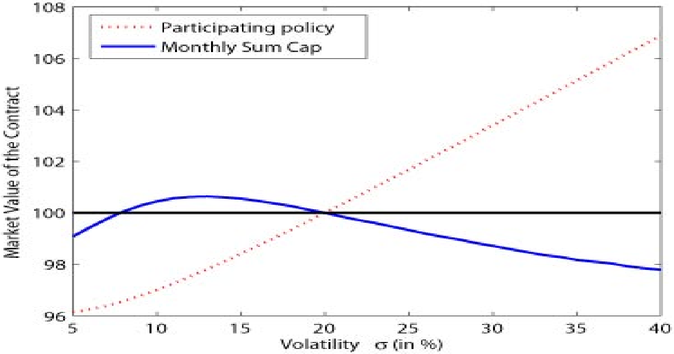
- Pay M in case of survival to time T.
- Pay M in case of death prior to T.

This will hedge "mortality risk" if the life expectancy increases or decreases for the whole population.

$\Rightarrow$ **Hedge of the systematic risk of the mortality risk**

Sensitivity of market values to the volatility σ

Sensitivity of the prices of Participating EIAs and Monthly Sum Caps to volatility. $r = 5\%$, $\mu = 0.09$, $\delta = 2\%$, maturity of $T = 1$ year. The participation is set at $k = 89.6\%$ and the monthly cap is equal to $c = 5.4\%$. Assuming $\sigma = 0.2$, the three contracts all have the same price of $100.



Natural Hedge for Sellers

Idea: The seller issues 100 policies:

- n Participating policies. The payoff is denoted by X_1.
- $100 - n$ Locally-capped contracts. The payoff is denoted by X_2.

$\mathcal{MV}(X, \sigma)$ is the market value at time 0 of the payoff X when the volatility is equal to σ in the Black and Scholes model. Consider

$$\mathcal{S}(\mathbf{n}) = \sup_{\sigma \in [\sigma_0 - \varepsilon, \sigma_0 + \varepsilon]} \mathbf{V}(\mathbf{n}, \sigma) - \inf_{\sigma \in [\sigma_0 - \varepsilon, \sigma_0 + \varepsilon]} \mathbf{V}(\mathbf{n}, \sigma)$$

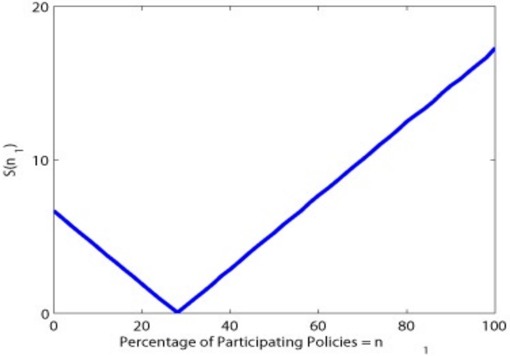
where $V(n, \sigma)$ is the market value of the portfolio of policies:

$$\mathbf{V}(\mathbf{n}, \sigma) = \mathcal{MV}(\mathbf{n}\mathbf{X_1} + (\mathbf{100} - \mathbf{n})\mathbf{X_2}, \sigma)$$

Let n^* be the number of contracts of type X_1, that minimizes $\mathcal{S}(n)$.

Natural Hedge for Sellers

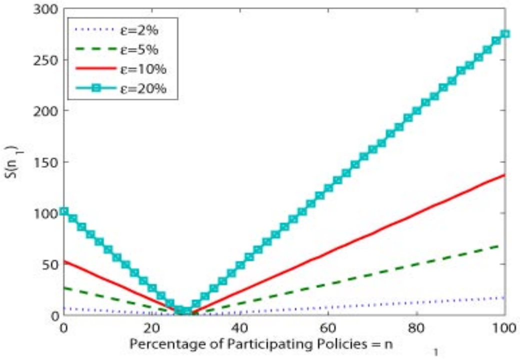
Assume $\varepsilon = 2\%$, $\sigma = 20\%$, $r = 5\%$, $\mu = 0.09$, $\delta = 2\%$, $g = 1\% p.a.$, $\sigma = 0.2$, $T = 1$ year with a monthly cap level equal to 5.4%. The participation rate is $k = 89.6\%$ and both contracts have a fair value equal to \$1.



The function $S(n)$ is minimized when the percentage of EIAs sold is equal to $n^* = 28$.

Natural Hedge for Sellers

Applied with different levels of ε to show that this measure is robust.

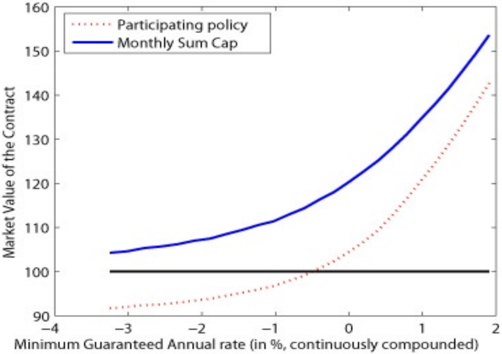


For each value of ε, the optimal percentage of EIAs is 28%.

- Typical insurance policies have annual guarantees (also called ratchet, step-up or cliquet option).
- Parameters
  - Maturity T years.
  - η is the minimum annual guaranteed rate (continuously compounded).
- Comparison with the case without annual guarantee.

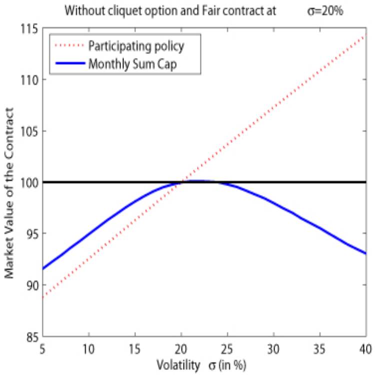
Cost of the Annual Guarantee

Both contracts are fairly priced (equal to \$100) without annual guarantee. $T = 5$ years, $r = 5\%$, $\delta = 2\%$, $\sigma = 20\%$, $\mu = 0.09$. The minimum guaranteed rate at maturity is $g = 2\%$ p.a.. The fair participating coefficient $k = 92.6\%$. The fair monthly cap level is 12.1%.



Increase sensitivity to volatility

$r = 5\%$, $\mu = 0.09$, $\delta = 2\%$, $g = 2\%$, $T = 5$ years. In panel A and in Panel B, assuming $\sigma = 0.2$, both contracts have the same price of \$100.
In Panel A, no annual guarantee, the fair participation $k = 92.6\%$, the monthly cap level $c = 12.1\%$.
In Panel B, annual minimum guaranteed rate of $\eta = 0\%$, the fair participation $k = 90.3\%$, the monthly cap level $c = 5.6\%$.



Natural hedge

- The sensitivity to volatility is amplified by the presence of an annual guarantee.
- Market values are therefore extremely sensitive to errors on the volatility parameter estimation.
- Natural hedge works similarly as the simple case.

Limitations and Future Work

- This is **only** a hedge of the balance sheet at time 0 against small changes in the volatility parameter / possible error in the estimation of the volatility.
  $\Rightarrow$ **It is not a dynamic hedge!** Need to consider what happens after $t = 0$ and if this natural hedge still holds.
- Assume the insurer delta hedges both types simultaneously, does it improve the efficiency of the dynamic hedging?
- These contracts are very sensitive to volatility. Black and Scholes model is not enough.
  $\Rightarrow$ Consider stochastic volatility models.