Timer-Style Options
Design, Pricing and Practice

RiO 2010

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Outline

- Realized volatility. What is a timer option?
- Model-free price when the risk-free rate is equal to 0.
- Some reasons for investing in timer options.
- Pricing timer option in general stochastic volatility models - Theoretical results.
- Pricing timer option in general stochastic volatility models - Numerical example.
- Further research on timer options.
- Other timer-style options and proposal for new timer options.

Discrete Realized Variance

- **Realized Variance** over $[0, T]$ (*discretely monitored*),

$$\Sigma^2_{realized} = \frac{1}{n-1}\sum_{i=1}^{n}\left(\ln\left(\frac{S_{t_i}}{S_{t_{i-1}}}\right)\right)^2$$

where $0 < t_1 < ... < t_n = T$

- **Annualized Realized Variance** over $[0, T]$ (*discretely monitored*),

$$\sigma^2_{realized} = \frac{\Sigma^2_{realized}}{T}.$$

- **Realized variance consumption.** As the stock moves daily. After d days, the **"variance budget"** is expended according to

$$VB^{realized} = \sigma^2_{realized}\frac{d}{252}.$$

Continuous Realized Variance

- Assume

$$\frac{dS_t}{S_t} = rdt + \sqrt{V_s}\left(\rho dW_s^1 + \sqrt{1-\rho^2}dW_s^2\right)$$

Realized variance consumption at T (**continuously monitored**) or quadratic variation of $\ln(S)$ over $[0, T]$.

$$\xi_T := \int_0^T V_s ds.$$

- Let $T = n\Delta$. The cumulative realized variance over $[0, T]$ is

$$\xi_T = \langle log(S) \rangle_T = \int_0^T (d \ln S_u)^2 = \lim_{n \to +\infty} \sum_{i=0}^{n-1} \left(\ln\left(\frac{S_{(i+1)\Delta}}{S_{i\Delta}}\right)\right)^2$$

- $(\xi_t)_{t \geqslant 0}$ may be viewed as a **stochastic clock**. We make use of "time change" techniques.

Modelling the financial market

- Assume a constant risk-free interest rate r. Under the risk neutral measure Q

$$\begin{cases} dS_t &= rS_t dt + \sqrt{V_t} S_t \left(\sqrt{1-\rho^2} dW_t^1 + \rho dW_t^2 \right), \\ dV_t &= \alpha_t dt + \beta_t dW_t^2 \end{cases}$$

where W^1 and W^2 are independent Brownian motions, and where α_t and β_t are adapted processes such that a unique solution (S_t, V_t) exists, $V_t > 0$ a.s. and

$$\xi_T = \int_0^T V_t dt$$

is well-defined and converges to $+\infty$ when $T \to +\infty$.

Perpetual Timer Options

A timer option is a standard option with random maturity.

- Denote by τ the **random** maturity time of the option.
- It is defined as the **first hitting time** of the realized variance to the variance budget $\mathbb{V}$

$$\tau = \inf\left\{u > 0, \int_0^u V_s ds = \mathbb{V}\right\} = \inf\left\{u > 0, \xi_u = \mathbb{V}\right\}.$$

- The payoff of a timer call option is paid at time τ and is

$$\max(\mathbf{S}_\tau - \mathbf{K}, \mathbf{0}).$$

Pricing timer options

- One can write

$$\ln(S_t) = \ln(S_0) + rt - \frac{1}{2}\xi_t + \int_0^t \sqrt{V_s}\left(\rho dW^2 + \sqrt{1-\rho^2}dW_1\right)$$

- Dubins Schwarz theorem applies and one gets

$$B_{\xi_t} = \int_0^t \sqrt{V_s}\left(\rho dW_s^2 + \sqrt{1-\rho^2}dW_s^1\right)$$

- One can write

$$\ln(S_t) = \ln(S_0) + rt - \frac{1}{2}\xi_t + B_{\xi_t}$$

- Then,

$$S_\tau = S_0 e^{r\tau} e^{B_{\xi_\tau} - \frac{1}{2}\xi_\tau} = S_0 e^{r\tau} e^{B_{\mathbb{V}} - \frac{1}{2}\mathbb{V}}.$$

Pricing timer options (cont'd)

Theorem

The initial price of a timer option is

$$C_0 = E^Q\left[\max\left(S_0 e^{B_{\mathbb{V}} - \frac{1}{2}\mathbb{V}} - Ke^{-r\tau}, 0\right)\right].$$

where $B_{\mathbb{V}} = B_{\xi(\tau)}$ *with* $B_u = \int_0^u \sqrt{V_t}\left(\sqrt{1-\rho^2}dW_t^1 + \rho dW_t^2\right)$ *is a Q-standard Brownian motion.*

Not an easy problem because τ and $B_{\mathbb{V}}$ are not independent and in general

$$B_{\mathbb{V}}|\tau$$

is not normally distributed.

Pricing timer options when $r = 0\%$

When $r = 0\%$, the price of a timer call option is

$$C_{0|r=0\%} = E^Q\left[\max\left(S_0 e^{B_{\mathbb{V}} - \frac{1}{2}\mathbb{V}} - K, 0\right)\right].$$

$\Rightarrow$ closed-form expression equal to the Black and Scholes formula with interest rate $r = 0\%$, volatility σ and maturity T that verify $\sigma^2 T = \mathbb{V}$, i.e. $C_{BS}\left(S_0, K, 0, \sqrt{\frac{\mathbb{V}}{T}}, T\right)$. Thus when $r = 0\%$, the price of a timer call is equal to

$$\mathbf{C_{0|r=0\%} = S_0 \mathcal{N}\left(\widehat{d}_1\right) - K\mathcal{N}\left(\widehat{d}_2\right)}$$

where $\widehat{d}_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \frac{1}{2}\mathbb{V}}{\sqrt{\mathbb{V}}}$ and $\widehat{d}_2 = \widehat{d}_1 - \sqrt{\mathbb{V}}$.

Implied Volatility

- Black and Scholes market

$$\frac{dS_t}{S_t} = rdt + \sigma dW_t$$

- Call option price $C_{BS}(S_0, K, r, \sigma, T)$

$$c(\sigma) := C_{BS}(S_0, K, r, \sigma, T) = S_0 \mathcal{N}(d_1) - Ke^{-rT}\mathcal{N}(d_2)$$

where $d_1 = \frac{\ln\left(\frac{S_0}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$.

- Market Price of the same call C_M observed in the market.
- σ_* **implied volatility** such that

$$c(\sigma_*) = C_M.$$

A simple observation

Let us compare when $r = 0\%$ a standard vanilla call with a perpetual timer call.

- Price at 0 of a standard call option

$$C_{BS}(S_0, K, \sigma_*, T)$$

  where σ_* is the implied volatility.

- Price at 0 of a timer call option

$$C_{BS}\left(S_0, K, \sqrt{\frac{\mathbb{V}}{T}}, T\right)$$

Since the Black and Scholes formula is an increasing function of the volatility, one only needs to compare $\mathbb{V}$ with $\sigma_*^2 T$.

Betting on Realized volatility vs Implied volatility

If one believes that the annualized realized volatility (over [0,T]) will be larger than the implied volatility at time T for a given option with a strike K then the following strategy is going to make money

- **Long a standard option**
- **Short a timer option**

We then investigate what can happen.

Case 1: If $\tau = T$, the two options have identical cash-flows.

Cash-flows in Case 2 ($\tau < T$)

If the realized variance $\xi_T > \sigma_*^2 T = \mathbb{V}$, then $\tau < T$. **P&L $\geqslant$ 0**!

- At time τ,
$$-(S_\tau - K)^+$$
- If it is 0 ($S_\tau \leqslant K$) then one makes strict profit by selling the remaining option at time $\tau < T$, the proceeds of the sale will be strictly positive.
- If it is negative ($S_\tau > K$) then one sells S and puts K in a bank account. At time T (since $r = 0\%$),
$$(S_T - K)^+ - S_T + K$$
If $S_T < K$, then strict profit of $K - S_T > 0$.
If $S_T \geqslant K$, then 0.

Cash-flows in Case 3 ($\tau > T$)

If the realized variance $\xi_T < \sigma_*^2 T = \mathbb{V}$, then $\tau > T$. **P&L $\leqslant$ 0**!

- At time T, one receives

$$(S_T - K)^+$$

- **If it is 0** ($S_T \leqslant K$) then one has a loss at T because the timer option is still alive with a positive premium.
- **If it is positive** ($S_T > K$). Then one would have a strict loss at time T. (The intuition is that the value of a call option is higher than its exercise value when the underlying does not pay dividends).

$$\textit{Value at } t > (S_t - K)^+.$$

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$$\textit{Value at } t > (S_t - K)^+.$$

To conclude, this strategy (short position in a timer option) guarantees **a sure profit when the realized volatility is higher than the implied volatility**! Why is this attractive?

Sawyer (2007) explains that "*this product is designed to give investors more flexibility and ensure they do not overpay for an option. The price of a vanilla call option is determined by the level of implied volatility quoted in the market, as well as maturity and strike price. But the level of implied volatility is often higher than realized volatility, reflecting the uncertainty of future market direction. [...] In fact, having analyzed all stocks in the Euro Stoxx 50 index since 2000, SG CIB calculates that 80% of three-month calls that have matured in-the-money were overpriced.*"

$\Rightarrow$ **A Long position in a timer option can make money in 80% of the time???**

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$\Rightarrow$ **A Long position in a timer option can make money in 80% of the time???**

Some references

Results and related discussions can be found in

- Mileage Options by Neuberger, A. (1990): *"Volatility trading,"* Working Paper, London Business School.
- *"Implied volatility, Realized volatility and Mileage options"* presented by Roger Lee at the Bachelier meeting, 2008. Joint work with P. Carr.
- Bick, A. (1995): *"Quadratic-Variation-Based Dynamic Strategies,"* Management Science, 41(4), 722–732.

where "perpetual timer options" are called "perpetual mileage options".

We assume that the variance process is now modeled by

$$\begin{cases} dV_t = \alpha(V_t)\,dt \;+\; \beta(V_t)\,dW_t^2 \\ dS_t = rS_t dt + S_t\sqrt{V_t}\left(\sqrt{1-\rho^2}dW_t^1 + \rho dW_t^2\right) \end{cases} \tag{1}$$

In the general stochastic volatility model given by (1),

$$\mathbf{S_T} = \mathbf{S_0}\exp\{\mathbf{rT} + \mathbf{a_T} + \sqrt{\mathbf{b_T}}\mathbf{Z}\},$$

where a_T and b_T are defined by

$$a_T = \rho(f(V_T) - f(V_0)) - \rho H_T - \frac{1}{2}\xi_T, \quad b_T = (1-\rho^2)\xi_T$$

with

$$H_T = \int_0^T h(V_t)dt \quad \text{and} \quad \xi_T = \int_0^T V_t dt$$

and where $Z \sim \mathcal{N}(0,1)$ independent of V_T, H_T and ξ_T and where f and h are defined by

$$f(v) = \int_0^v \frac{\sqrt{z}}{\beta(z)}dz, \quad h(v) = \alpha(v)f'(v) + \frac{1}{2}\beta^2(v)f''(v) \tag{2}$$

Standard European Options

Theorem

Standard European Call Option.

The price of a standard call option with maturity T is equal to

$$E\left[C_{BS}(\widehat{S}_0, K, r, \widehat{\sigma}, T)\right]$$

where C_{BS} is the Black Scholes call price with $\widehat{S}_0 = S_0 \exp\left(a_T + \frac{b_T}{2}\right)$ and $\widehat{\sigma} = \sqrt{b_T}$. Note that a_T and b_T depend on (V_T, ξ_T, H_T) where $\xi_T := \int_0^T V_s ds$ and $H_T = \int_0^T h(V_s)ds$.

Therefore,

$$Call\ price = E\left[\Psi_1\left(V_T, \xi_T, H_T\right)\right].$$

Perpetual Timer Options

Theorem

Timer Call Option.
In a general stochastic volatility model given by (1), the price of a timer call option can be calculated as

$$C_0 = E\left[S_0 e^{a_\tau + \frac{(1-\rho^2)\mathbb{V}}{2}}\mathcal{N}(d_1) - Ke^{-r\tau}\mathcal{N}(d_2)\right] \tag{3}$$

where $a_\tau = \rho(f(V_\tau) - f(V_0)) - \rho H_\tau - \frac{1}{2}\mathbb{V}$ *and*

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right)+r\tau+a_\tau+(1-\rho^2)\mathbb{V}}{\sqrt{(1-\rho^2)\mathbb{V}}}, \quad d_2 = d_1 - \sqrt{(1-\rho^2)\mathbb{V}}.$$

$$Timer\ \ Call\ \ Price = E\left[\Psi_2\left(\tau, V_\tau, H_\tau\right)\right]\ \ with\ H_\tau = \int_0^\tau h(V_s)ds.$$

Perpetual Timer Options ($\rho = 0$)

Theorem

When $\rho = 0$, the price of a timer call option in a general stochastic volatility model is given by

$$C_{0|\rho=0} = S_0 E^Q\left[\mathcal{N}\left(d_1(\tau)\right)\right] - KE^Q\left[e^{-r\tau}\mathcal{N}\left(d_2(\tau)\right)\right] \quad (4)$$

where

$$d_1(\tau) = \frac{\ln\left(\frac{S_0}{K}\right) + \frac{1}{2}\mathbb{V} + r\tau}{\sqrt{\mathbb{V}}}, \quad d_2(\tau) = d_1(\tau) - \sqrt{\mathbb{V}}.$$

$$Timer\ \ Call\ \ Price = E\left[\Psi_3\left(\tau\right)\right]$$

Recall that

$$\tau := \tau(\mathbb{V}) = \inf\left\{t; \int_0^t V_s ds = \mathbb{V}\right\} \in (0, \infty)$$

is the first passage time of the integrated functional of V_s to the fixed level $\mathbb{V} \in (0, \infty)$, then the law of (τ, V_τ, H_τ) is given by

$$(\tau, \mathbf{V}_\tau, \mathbf{H}_\tau) \overset{\text{law}}{\sim} \left(\int_0^{\mathbb{V}} \frac{\mathbf{1}}{\mathbf{X_s}}\mathbf{ds},\ \mathbf{X}_{\mathbb{V}},\ \int_0^{\mathbb{V}} \frac{\mathbf{h(X_s)}}{\mathbf{X_s}}\mathbf{ds}\right) \tag{5}$$

where X_t is governed by the SDE

$$\begin{cases} df(X_t) & = & \frac{h(X_t)}{X_t}dt + dB_t, \\ X_0 & = & V_0 \end{cases} \tag{6}$$

where B is a standard Brownian motion, and f and h are given by (2).

Monte Carlo Approach

We will make use of the main result to simulate an i.i.d. sample of (τ, V_τ, H_τ). The estimate of the timer call option price is obtained by Monte Carlo

$$Timer\ \ Call\ \ Price = E\left[\Psi_2\left(\tau, V_\tau, H_\tau\right)\right].$$

$$Timer\ \ Call\ \ Price\ \ when\ \ \{\rho = 0\} = E\left[\Psi_3\left(\tau\right)\right].$$

Variance Reduction

Using the model-free closed-form expression $C_{0|r=0\%}$ for the timer option given in a general stochastic volatility model, it is possible to significantly improve the convergence of the Monte Carlo estimator. We estimate the price by

$$C_0^{mc} - \lambda\left(\widetilde{C}_0 - C_{0|r=0\%}\right) \tag{7}$$

where

$$\widetilde{C}_0 = \frac{S_0 e^{\frac{(1-\rho^2)\mathbb{V}}{2}}}{n}\sum_{i=1}^{n} e^{a_{\tau_i}}\mathcal{N}\left(d_1(\tau_i, V_{\tau_i}, H_{\tau_i})\right) - \frac{K}{n}\sum_{i=1}^{n}\mathcal{N}\left(d_2(\tau_i, V_{\tau_i}, H_{\tau_i})\right),$$

where $\lambda = corr(C_0^{mc}, \widetilde{C}_0)$ and where a_{τ_i}, d_1 and d_2 are defined in (3).

Numerical Example

Numerical examples are derived in the Heston model and in the Hull and White model.
In the Heston model,

$$dV_t = \kappa\left(\theta - V_t\right)dt + \gamma\sqrt{V_t}dW_t^2$$

where κ, θ and γ are the parameters of the volatility process, they are all positive and the Feller condition $2\kappa\theta - \gamma^2 > 0$ ensures that $V_t > 0$ a.s.. In the Hull and White model,

$$dV_t = aV_t dt + \nu V_t dW_t^2$$

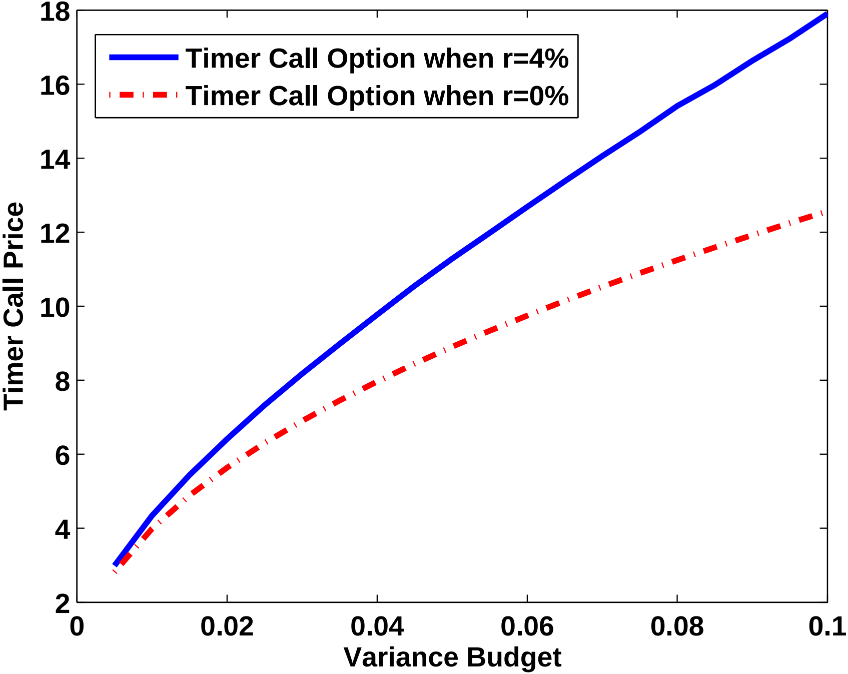
where a and ν are positive.

Parameters in the Heston Model

Example in the Heston model

$S_0 = 100$	$K = 100$	$r = 0.04$	$\mathbb{V} = 0.0265$
$\kappa = 2$	$V_0 = 0.0625$	$\gamma = 0.1$	$\theta = 0.0324$

This graph is obtained with a time step of $1/3000$ and $1,000,000$ Monte Carlo simulations for each value of the variance budget "$\mathbb{V}$". The correlation is $\rho = -0.5$.


Timer Call Option when r=4%
Timer Call Option when r=0%
Timer Call Price
Variance Budget
18
16
14
12
10
8
6
4
2
0
0.02
0.04
0.06
0.08
0.1

The transformation we do is good because it allows us to get a much better convergence.

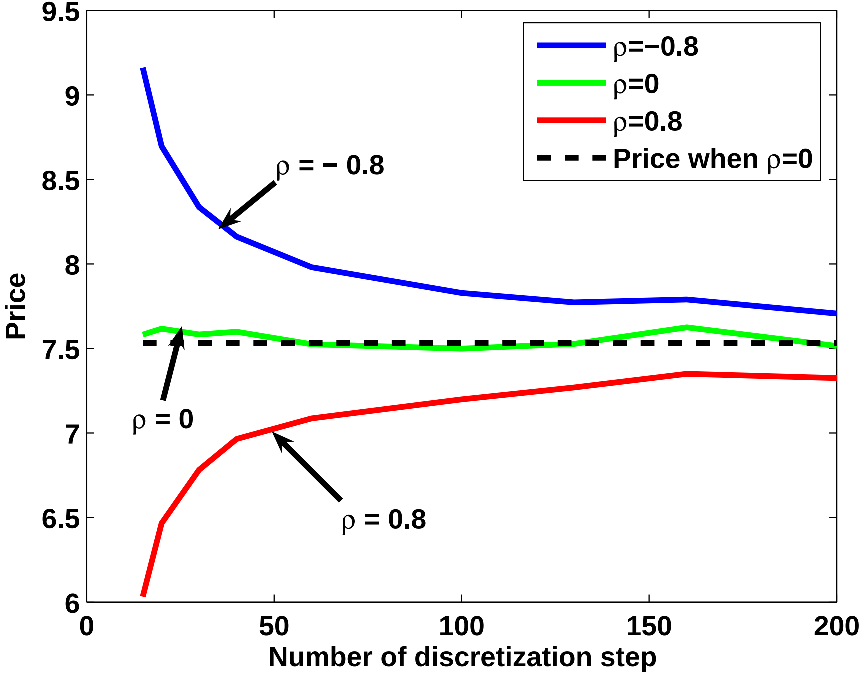
- **First Figure:** direct simulation of (S_t, V_t) to get the price of a timer option by Monte Carlo as

$$E[e^{-r\tau}(S_\tau - K)^+]$$

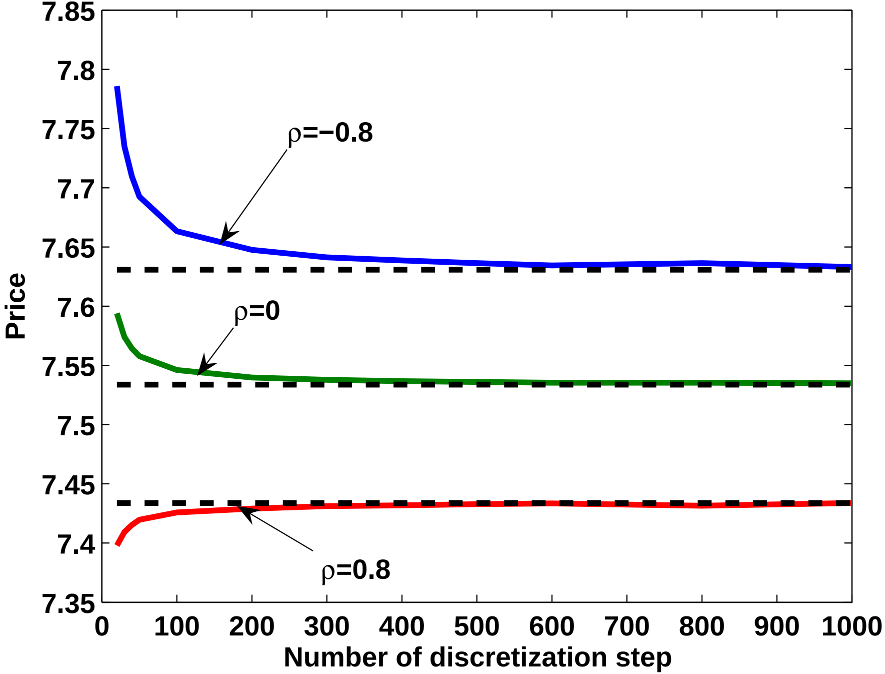
  The number in parenthesis are the standard deviations obtained with 100,000 simulations.

- **Second Figure:** implementing our techniques using 50,000 simulations for each value of M.

Pay attention to the scales of the y-axis.



ρ=−0.8
ρ=0
ρ=0.8
Price when ρ=0
ρ = − 0.8
ρ = 0
ρ = 0.8
Price
Number of discretization step
9.5
9
8.5
8
7.5
7
6.5
6
0
50
100
150
200



7.85
7.8
7.75
7.7
7.65
7.6
7.55
7.5
7.45
7.4
7.35
Price
0
100
200
300
400
500
600
700
800
900
1000
Number of discretization step
ρ=−0.8
ρ=0
ρ=0.8

Further issues

Specific questions related to the timer options

- Which scheme is efficient to similate the Bessel process underlying the price of a timer option?
- Can we use results on Bessel processes to improve the pricing and hedging of timer options?

More general questions

- What about if the realized variance is discretely monitored instead of continuously monitored?
- What about the effect of stochastic interest rates and correlation between interest rates and volatility?

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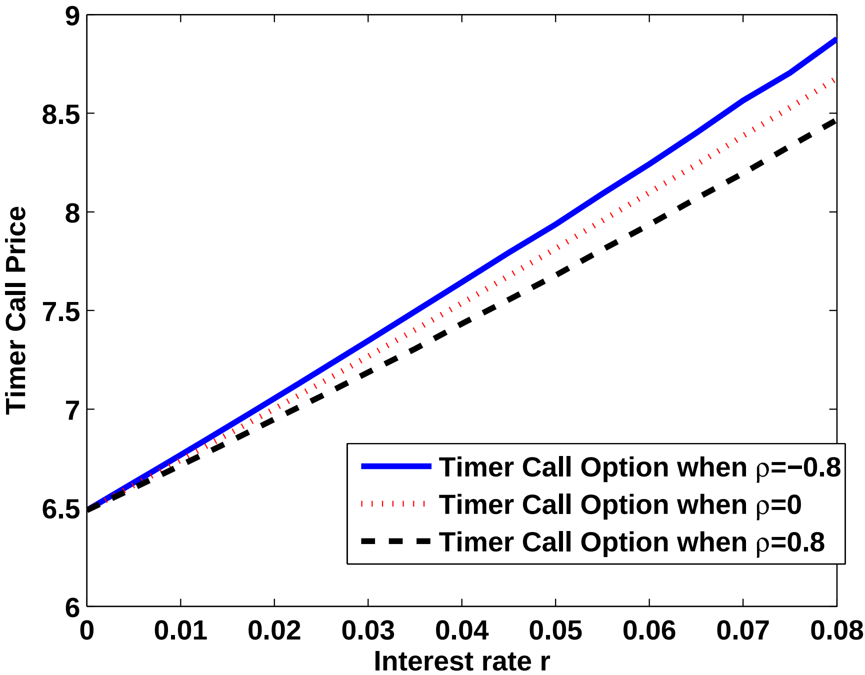

Timer Call Price
Interest rate r
9
8.5
8
7.5
7
6.5
6
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
Timer Call Option when ρ=−0.8
Timer Call Option when ρ=0
Timer Call Option when ρ=0.8

Finite Expiry Timer Options

How to value a timer option with expiry

$$\min(\tau, T)?$$

How large is the difference relative to the perpetual timer options?

$$E_Q[e^{-r\tau}\mathbb{1}_{\tau<T}(S_\tau - K)^+ + e^{-rT}\mathbb{1}_{\tau\geq T}(S_T - K)^+].$$

Finite Expiry Timer Options

A model-free upper bound can be derived

$$E_Q\left[e^{-r\tau\wedge T}(S_{\tau\wedge T}-K)^+\right] \leqslant \min\left(E_Q\left[e^{-r\tau}(S_\tau-K)^+\right], E_Q\left[e^{-rT}(S_T-K)^+\right]\right)$$

"Implied volatility, Realized volatility and Mileage options" presented by Roger Lee at the Bachelier meeting, 2008. Joint work with P. Carr.

FX Timer options

A FX timer option is very similar to a timer call. The only difference lies in the underlying process. In a FX timer option, the investor sets a variance budget for the exchange rate and it is a call option on the **exchange rate** ("timer caplet"). When the accumulated variance of the exchange rate has reached the budget, the caplet expires.

Time Swap (description)

Sawyer (2007). *"The time swap, on the other hand, gives investors a means to short volatility with an inverted convexity profile (meaning the downside is limited). Rather than volatility, the strike is expressed in days. In other words, if an investor wants to short volatility over a specified investment period - for instance, three months - a variance budget is calculated with a volatility level set by SG CIB. The payout is based on the number of days required to consume the variance budget minus the specified investment horizon, times the notional. So, if realised volatility is consistently lower than the specified level, it will take longer than three months for the option to expire, and the investor receives a payout."*

Time Swap (Modelling)

- Compare the **target expiry** time T with the **random** time τ defined as
$$\tau = \inf\{u > 0, \int_0^u V_s ds = \mathbb{V}\}.$$
- As a standard swap, a time swap has a notional amount, K.
- At τ when the variance budget is expended, the payoff is
$$K(\tau - T).$$
Then, the price of the time swap is given as follows
$$KE\left[e^{-r\tau}(\tau - T)\right].$$
- Some properties of a long position
  - positive payoff if $\tau > T$ and negative when $\tau < T$.
  - short position in realized volatility.
  - limited exposure (maximum loss is KT)

Timer out-performance option

The timer out-performance option was developed shortly after the first timer call option was sold in April 2007. Sawyer explains that *"the out-performance product is similar to the timer call the investor specifies a target volatility for the spread between two underlyings and a target investment horizon, which is used to calculate a variance budget. Mattatia claims the timer out-performance call can be 30 percent cheaper than a plain vanilla out-performance option "*

Timer out-performance option (modelling)

- Consider two correlated assets S_1 and S_2.
- Specify a target volatility σ_0 for the spread between the two underlying's log-return and a target investment horizon T.
- A variance budget is calculated as

$$\mathbb{V} = \sigma_0^2 T.$$

Define then

$$\tau = \inf\left\{u > 0, \left\langle \ln \frac{S_2}{S_1} \right\rangle_u = \mathbb{V}\right\}.$$

- Payoff at τ of the timer out-performance option

$$\max\left(S_2(\tau) - S_1(\tau), 0\right).$$

Timer-style options

- Forward start timer option
- Compound timer option
- Timer cliquet option
- Timer barrier option
- Timer Lookback option
- ...

Discretization is not uniformly done but done along a random grid.

- Bernard C. and Cui Z. (2010): "Pricing Timer Options", Journal of Computational Finance, forthcoming.
- Carr, P., and Lee, R. (2009): "Volatility Derivatives," Annual Review of Financial Economics, 1, 319–339.
- Carr, P., and Lee, R. (2010): "Hedging Variance Options on Continuous Semimartingales," Finance & Stochastics, 14, 179–207.
- Lee, R. (2008): "Implied volatility, Realized volatility and Mileage options", Bachelier meeting, 2008. Joint work with P. Carr.
- Li, C.X. (2010): "Managing Volatility Risk," PhD thesis, Columbia University.
- Neuberger, A. (1990): "Volatility trading," Working Paper, London Business School.
- Sawyer, N. (2007): "SG CIB Launches Timer Options," RISK magazine, 20(7).
- Sawyer, N. (2008): "Equity Derivatives House of the Year - Société Générale - Risk Awards 2008," RISK magazine, 21(1).
- Société Générale Corporate & Investment Banking (2007): "The Timer Call: A Floating Maturity Option," Power Point.