Natural Balance Sheet Hedge of Equity Indexed Annuities

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Introduction

Equity Linked Insurance Market

- Contracts sold by insurance companies (Variable Annuities, Equity Indexed Annuities, Unit-linked contracts...)
- They usually provide a complicated payoff related to some reference portfolio. The payoff design can be modified and extended in countless ways. Here are some of them:
  - Guaranteed floor (periodically or at maturity)
  - Upper limits or caps
  - Path-dependent payoffs (Asian, lookback, barrier), locally-capped contracts and cliquet options
  - Embedded complex life benefits: GMXB
- They have become very popular in many countries (the total VA assets in the US were $1.41 trillion as of June 30, 2008.)
Current Economic Context:

- **New regulation and new accounting standards** (proposed by the IASB (International Accounting Standards Board) in Europe and by the FASB (Financial Accounting Standards Board) in the US.

- **“fair value” or “mark-to-market”** reporting system: Insurers are required to evaluate EIAs at their market value in their balance sheet

- Europe, US, Australia and Asia are adopting or about to adopt such systems.

However such change in the regulation is highly **controversial**...
Controversial Change


► **positive** because

- “the market value of a liability is more relevant than historical cost... it reflects the amount at which that liability could be incurred or settled in a current transaction between willing parties.”
- More transparency.

► **negative** because

- “market values” cannot be obtained if there exists no actual liquid market.
- market values increase the volatility of the annual results of companies and is contrary to the smooth return policyholders and shareholders would prefer.
- reporting standards might induce excessive volatility in the markets.
Many Interesting Issues about EIAs

► **Pricing, hedging and risk management.** Market values.

► **Design** from buyers’ perspective (*choice of the right (optimal) contract to buy*).

► **Design** from insurers’ perspective (*choice of the right portfolio of policies to sell*).
   
   - We show **how to stabilize aggregate liabilities market value by building a portfolio of policies.**
   - Insurers can immunize their balance sheet against market changes and parameter uncertainty by carefully combining different payoffs.
Outline of the paper

- Description of common contracts
- Natural Hedge of volatility risk.
- Effects of embedded ratchet options or annual guarantee.
Two popular designs

Initial investment = $M

We focus on two popular designs sold by insurance companies:

- **Standard Equity Indexed Annuities** (participating policy) with payoff given by:

\[ X_T = M \max \left( e^{gT}, k \frac{S_T}{S_0} \right) \]

where \( k \) is called the participating rate and \( g \) stands for the minimum guaranteed rate at maturity.

- Periodically-capped contracts. Ex: **Monthly Sum Cap** with cap level equal to \( c \) on the return of each month.
Monthly Sum Cap

- Initial investment = $M
- Minimum guaranteed rate \( g \) at maturity \( T \) years.
- Local Cap \( c \) on the monthly return.
- Let \( t_0 = 0, t_1 = \frac{1}{12}, t_2 = \frac{2}{12}, \ldots, t_n = \frac{n}{12} = T \). The payoff \( Z_T \) of the monthly sum cap is linked to

\[
\sum_{i=1}^{n} \min \left( c, \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}} \right)
\]
### Monthly Sum Cap \((c = 3\%), \ T = 1\) year, Year 2003.

<table>
<thead>
<tr>
<th>Month</th>
<th>Raw S&amp;P return</th>
<th>Adjusted Return used for Monthly Sum Cap</th>
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<tr>
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<tr>
<td>12</td>
<td>5.07</td>
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The sum of the adjusted returns in the third column is 12.45\%.
Monthly Sum Cap \((c = 3\%), \ T = 1\ \text{year}, \ Year\ 2008\).

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<th>Adjusted Return used for Monthly Sum Cap</th>
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The sum of the adjusted returns in the third column is \(-47.2\%).
Monthly Sum Cap Contract

- Initial investment = $M$
- Minimum guaranteed rate $g$ at maturity $T$ years.
- Local Cap $c$ on the monthly return.
- Let $t_0 = 0$, $t_1 = \frac{1}{12}$, $t_2 = \frac{2}{12}$, ..., $t_n = \frac{n}{12} = T$. The payoff $Z_T$ of the monthly sum cap contract is

$$Z_T = M \max \left( e^{gT}, 1 + \sum_{i=1}^{n} \min \left( c, \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}} \right) \right)$$

- The contract consists of:
  - a zero-coupon bond
  - a complex option component

Pricing by Monte Carlo or by Fast Fourier analysis.
Natural Hedge for Insurers

What is a “natural hedge”? 

Well-known example, to hedge mortality risk, life insurance companies can offer simultaneously two types of policies to people in the same age class:

• Pay $M$ in case of survival to time $T$.
• Pay $M$ in case of death prior to $T$.

This will hedge “mortality risk” if the life expectancy increases or decreases for the whole population.

⇒ Hedge of the systematic risk of the mortality risk
Sensitivity of market values to the volatility $\sigma$

Sensitivity of the prices of Participating EIAs and Monthly Sum Caps to volatility. $r = 5\%$, $\mu = 0.09$, $\delta = 2\%$, maturity of $T = 1$ year. The participation is set at $k = 89.6\%$ and the monthly cap is equal to $c = 5.4\%$. Assuming $\sigma = 0.2$, the three contracts all have the same price of $100$. 

![Graph showing the sensitivity of market values to volatility](image-url)
Natural Hedge for Sellers

Idea: The seller issues 100 policies:

- \( n \) Participating policies. The payoff is denoted by \( X_1 \).
- \( 100 - n \) Locally-capped contracts. The payoff is denoted by \( X_2 \).

\( \mathcal{MV}(X, \sigma) \) is the market value at time 0 of the payoff \( X \) when the volatility is equal to \( \sigma \) in the Black and Scholes model. Consider

\[
S(n) = \sup_{\sigma \in [\sigma_0 - \epsilon, \sigma_0 + \epsilon]} \mathcal{V}(n, \sigma) - \inf_{\sigma \in [\sigma_0 - \epsilon, \sigma_0 + \epsilon]} \mathcal{V}(n, \sigma)
\]

where \( \mathcal{V}(n, \sigma) \) is the market value of the portfolio of policies:

\[
\mathcal{V}(n, \sigma) = \mathcal{MV}(nX_1 + (100 - n)X_2, \sigma)
\]

Let \( n^* \) be the number of contracts of type \( X_1 \), that minimizes \( S(n) \).
Natural Hedge for Sellers

Assume $\varepsilon = 2\%$, $\sigma = 20\%$, $r = 5\%$, $\mu = 0.09$, $\delta = 2\%$, $g = 1\% p.a.$, $\sigma = 0.2$, $T = 1$ year with a monthly cap level equal to $5.4\%$. The participation rate is $k = 89.6\%$ and both contracts have a fair value equal to $\$1$.

The function $S(n)$ is minimized when the percentage of EIAs sold is equal to $n^* = 28$. 

Carole Bernard
Natural Hedge for Sellers

Applied with different levels of $\varepsilon$ to show that this measure is robust.

For each value of $\varepsilon$, the optimal percentage of EIAs is 28%.
Typical insurance policies have annual guarantees (also called ratchet, step-up or cliquet option).

Parameters
- Maturity $T$ years.
- $\eta$ is the minimum annual guaranteed rate (continuously compounded).

Comparison with the case without annual guarantee.
Cost of the Annual Guarantee

Both contracts are fairly priced (equal to $100) without annual guarantee. $T = 5$ years, $r = 5\%$, $\delta = 2\%$, $\sigma = 20\%$, $\mu = 0.09$. The minimum guaranteed rate at maturity is $g = 2\%$ p.a.. The fair participating coefficient $k = 92.6\%$. The fair monthly cap level is 12.1%.
Increase sensitivity to volatility

$r = 5\%, \mu = 0.09, \delta = 2\%, g = 2\%, T = 5$ years. In panel A and in Panel B, assuming $\sigma = 0.2$, both contracts have the same price of $100$.

In Panel A, no annual guarantee, the fair participation $k = 92.6\%$, the monthly cap level $c = 12.1\%$.

In Panel B, annual minimum guaranteed rate of $\eta = 0\%$, the fair participation $k = 90.3\%$, the monthly cap level $c = 5.6\%$. 
Natural hedge

- The sensitivity to volatility is amplified by the presence of an annual guarantee.

- Market values are therefore extremely sensitive to errors on the volatility parameter estimation.

- Natural hedge works similarly as the simple case.
Limitations and Future Work

- This is **only** a hedge of the balance sheet at time 0 against small changes in the volatility parameter / possible error in the estimation of the volatility.
  \[ \Rightarrow \text{It is not a dynamic hedge!} \] Need to consider what happens after \( t = 0 \) and if this natural hedge still holds.

- Assume the insurer delta hedges both types simultaneously, does it improve the efficiency of the dynamic hedging?

- These contracts are very sensitive to volatility. Black and Scholes model is not enough.
  \[ \Rightarrow \text{Consider stochastic volatility models.} \]